

ON FORECASTING THE SUNSPOT NUMBERS

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ABSTRACT. We have applied a technique recently proposed basing on learning nonlinear dynamics locally to describe the annual sunspot relative numbers. It is proved that the number of past points for prediction should be greater than 4 but less than 10. This rather simple approach yields in average relatively good results for short-term forecasts (< 11 yr). Particularly, it predicts that the current cycle no. 22 will reach a very high maximum. However, this approach must be modified in the vicinity of a grand minimum.

1. MOTIVATION

The problem of forecasting the solar activity is a classical one (cf. Witinskij et al., 1986). Since dynamo models of this process have not been developed on the quantitative level, one has to make predictions directly from the data. Yule (1927) was the first to predict the sunspot numbers using a stochastic autoregressive model. Further attempts viewed the Wolf numbers as a realization of a random process superimposing trends and "seasonal" components (Sneyers and Cugnon, 1986 or Wilson, 1988).

In the last decade a lot of examples in nonlinear dynamics has indicated that irregular behaviour is a manifestation of low-dimensional chaos (Ruelle, 1989). Due to the exponential instability of initially nearby trajectories chaos places a fundamental limit on long-term prognoses, but short-term forecasts might be quite accurate because such systems are deterministic.

Applying an attractor analysis to the Wolf numbers, some evidence to low-dimensional nonlinear dynamics have been yielded (Kurths, 1987, Ajmanova and Makarenko, 1988, Morfill and Voges, 1989). Basing on these results we deal the problem of forecasting the solar activity in general

and we make predictions of the Wolf numbers for the present solar cycle no. 22.

2. THE METHOD

2.1. Phase Space Reconstruction

The state of the dynamical system underlying the solar activity is determined by several variables. Their number corresponds to the attractor dimension which has been estimated less than 3 from an analysis of the Wolf numbers $W(t)$. It is possible to reconstruct the dynamical variables from this single time series by using Takens' (1981) proposal of delay-coordinate to form a vector

$$\underline{w}(t) = (W(t), W(t+\tau), \dots, W(t+(m-1)\tau)) \quad (1)$$

where m is the integer number determining the space in which the attractor is embedded.

2.2. Forecasting Model

As was shown by Farmer and Sidorovich (1987) a relationship between a current state and its future

$$\underline{w}(t+\Delta t) = f(\underline{w}(t))$$

over a short prediction time Δt can be approximated by a linear law

$$\underline{w}(t+\Delta t) = \underline{a} \underline{w}(t) + \underline{b}. \quad (2)$$

The constant matrix \underline{a} and the vector \underline{b} can be calculated applying the method of least squares. It is clear that $k > m+1$ points at $t' < t$, which are the nearest spatial neighbours of $\underline{w}(t)$, are needed for this approximation.

3. FORECASTING THE WOLF NUMBERS

In this paper we take the annual sunspot relative numbers from 1749 to 1988. The reversals of the magnetic polarity between subsequent cycles are taken into account by giving alternating signs to successive cycles (Fig. 1), as proposed by Bracewell (1986).

From the attractor analyses (Kurths, 1987, Ajmanova and Makarenko, 1988, Morfill and Voges, 1989) we have yielded that an embedding dimension $m = 3-4$ and a delay time $\tau = 2-5$ yr are suitable parameters to embed $W(t)$ via (1). Next, we have to choose $k > m+1$ (in fact $k = 5-8$ is

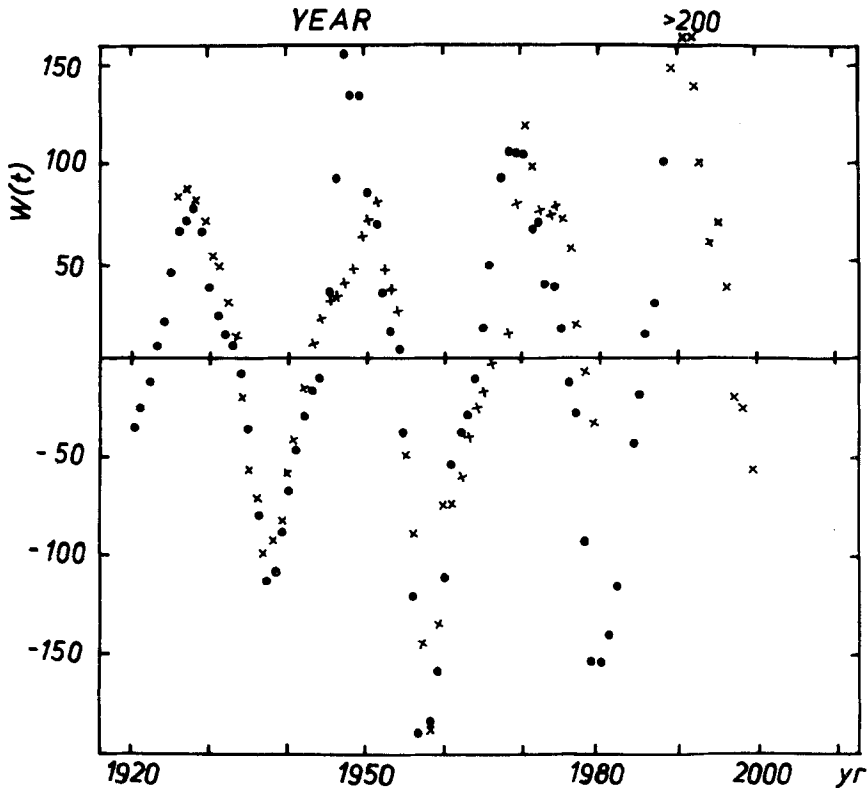


Fig. 1. A part of the annual relative sunspot numbers (dots) and some of their forecasts (crosses) basing on eq. 2

sufficient) spatial neighbours of various reference points and estimate the model parameters \underline{a} and \underline{b} for each reference point as well as for different prediction times Δt .

The first 175 points (1749 - 1923) are used as a "learning" sample to fit the matrices \underline{a} resp. the vectors \underline{b} in eq. 2, i. e. we "learn" the dynamics inherent the Wolf numbers from this part.

4. RESULTS

This method provides relatively good predictions (Fig. 1) which reproduce low as well as high maxima in the case of rather short-term forecasts ($\Delta t < 10$ yr). Note that this

procedure assumes homogeneity of the solar attractor. Indeed, this attractor appears to be inhomogeneous meaning, loosely speaking, that a trajectory visits some parts of the attractor more frequently than others. The influence of this fact to the forecast will be studied in a forthcoming paper with Makarenko and Ajmanova.

Applying this technique to a short-term prognosis, we yield indications that the present cycle no. 22 will produce a very high maximum. These results give further support to a chaotic behaviour of the solar activity which precludes long-term predictions. This is the reason why for longer predictions considerable errors in both amplitude and phase occur.

The pronounced manifestations of chaotic behaviour are grand minima like the Maunder minimum occurred over 70 yr in the 17th century (Eddy, 1983). In order to forecast the solar activity in the vicinity of a grand minimum one should take into account long-term characteristics of the dynamics, too.

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