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AN ATTRACTOR IN A SOLAR TIME SERIES

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A time series analysis of observed solar radio pulsations suggests that there must be a low-dimensional attractor. The power spectrum cannot be interpreted as a superposition of periodic components. Estimates of the maximum Lyapunov exponent and of the Kolmogorov entropy give some indications for a deterministic chaos. In order to study the limitations inherent in small data samples we include data from the Lorenz model and an artificial noise record. Consequences for the physical modelling of the pulsation event are discussed.

1. Introduction

Geophysical and astrophysical research rely on experimental data. In contrast to other fields of physics it is generally not possible to influence the medium under consideration or to repeat an observation under the same conditions. Nevertheless, measurements play an important role in modelling the physical processes involved.

Usually, the correlation analysis and the spectral analysis are applied to the data [1], which is connected with the physical concept of mode in linear theories. New methods for the description of dynamical systems (in particular of the deterministic chaos) have been developed. The concept of mode has been supplemented by other quantities such as dimension, Lyapunov exponents and the Kolmogorov entropy. Recently, algorithms have been proposed to estimate these quantities from measurements [2].

Here, we report observations of solar radio radiation exhibiting a pulsating structure, which is a specific manifestation of solar activity. The present paper deals with these measurements. Both concepts of time series analysis are used to obtain some aids for modelling the physical background mechanism.

In section 2 we introduce the data. Section 3 deals with the spectral analysis. Estimates of the attractor dimension are treated in section 4. Further basic properties of an attractor, such as the maximum Lyapunov exponent and the Kolmogorov entropy, are discussed in section 5. Section 6 contains an interpretation of our results.

2. Observations

The variation of the solar electromagnetic radiation and the particle emission is mainly caused by solar active regions. Solar radio emission in the dm–m wave range provides information about the physical processes in the chromosphere and the corona. Flares are the most violent manifesta-

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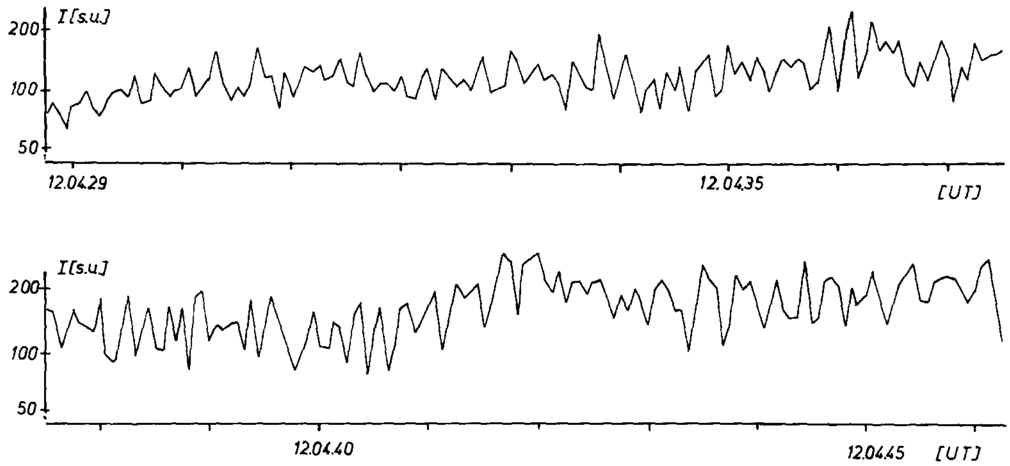


Fig. 1. The radio flux record of the pulsating structures on June 3, 1982 at the frequency of 778 MHz obtained at the Tremsdorf observatory (first part). The density I of the solar flux is given in Solar Units (1 S. U. = 10^{-22} W m^{-2} Hz^{-1}).

tion of solar activity. They are typically associated with an impulsive energy release [3]. It should be mentioned that research still has not led to a satisfactory model to describe the flare mechanism. Monitoring the process of the generation and propagation of radio waves in the Sun's atmosphere is complicated, too.

On June 3, 1982, an impulsive solar flare was observed. The main phase of the radio flare was a so-called type-IV burst with pulsating structures of the radio flux in the frequency range of 480–800 MHz [4]. Usually, the duration of such a solar radio pulsation event is not longer than a minute. The duration of the considered pulsations (40 s) provides a data length of 640 points, which refers to a sampling rate of $\Delta t = 0.064$ s. In the following, results are related to this time unit Δt. The record obtained by a single frequency radiometer at the Tremsdorf observatory is partly shown in fig. 1.

Models for solar radio pulsations are based on the linearized equations of MHD oscillations or on the time evolution of plasma instabilities. Often, these approximations explain the pulsations as a sum of periodic modes (peaks in the power spectrum) [5, 6].

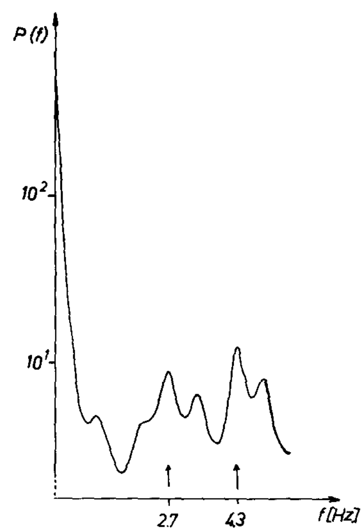


Fig. 2. The power spectrum of the whole solar record (see fig. 1).

3. Spectral analysis

A stationary stochastic process can be described by its power spectrum (Fourier transform of the autocovariance function) [7]. There are indications that during pulsations a relative stationarity may be assumed. The autocovariance function estimated from the record decreases with a correlation time of about ten. The estimation of the maximum entropy spectrum (MES), as proposed by Burg, provides a resolution which is much higher than is usually obtained by other methods [8]. The inspection of the MES computed from the considered data shows a broad-band power spectrum (fig. 2). The peaks are inside an 80% confidence interval. Hence, we cannot call them significant.

For comparison, we study autoregressive (AR) approaches which cover a wide range of stochastic processes. They are defined by the stochastic difference equation

$$X_t = \sum_{k=1}^{p} a_k X_{t-k} + N_t, \tag{1}$$

where N_t is identically and independently distributed (white noise). From eq. (1) it follows immediately that the power spectrum of such an AR-process can be written as

$$P_{XX}(f) = \frac{\sigma_N^2}{\left|1 - \sum_{k=1}^{p} a_k \exp(-2\pi i f k)\right|^2} \tag{2}$$

(σ_N^2 – variance of N_t) [9]. From eq. (2) we easily find a regular 5th order AR-process whose power spectrum (fig. 3) is very similar to that of the solar record (fig. 2). But in contrast to this an AR-model adaption does not yield a satisfactory description of the data in the time domain, i.e. the structure of the solar record is different from AR-processes in spite of their spectral resemblance.

For a generalization of the above analysis we have applied a time-dependent parameter estimate, too (sliding algorithms) [10], which likewise does not contribute to a sufficient reduction of the model errors.

Summarizing, we cannot determine significant structural parameters of the record using methods of the spectral analysis only.

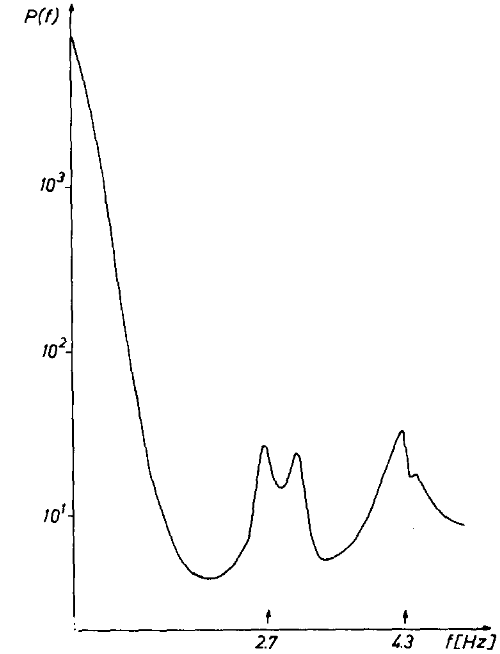


Fig. 3. The power spectrum of an AR-process of the 5th order.

Therefore, further investigations are devoted to a new field of time series analysis concerning the dimension of a possible attractor and a stability analysis in order to distinguish between quasi-periodicity, deterministic chaos and random noise.

In view of the rather short record (640 points) (cf. [11–14]) we treat typical models of the same length in order to check the reliability of our results:

a) AR-5 process ($p = 5$ in eq. (1), fig. 3);
b) periodic data;
c) the Lorenz model [15] with $\Delta t = 0.1$;

$$\frac{dx}{dt} = -10(x - y),$$
$$\frac{dy}{dt} = x(28 - z) - y, \tag{3}$$
$$\frac{dz}{dt} = xy - \frac{8}{3}z.$$

Now we can test the applicability of the algorithms to such short data lengths.

4. Attractor dimension

We focus our attention on the question: Is there a low-dimensional attractor underlying the observed pulsations?

The dimension furnishes an estimate of the number of variables which are at least required to describe the system. For the study of complex attractors the concept of fractal dimensions is useful [16].

We consider three generalized order-q dimensions [12]:

D_0 –capacity or fractal dimension;

D_1 –information or pointwise dimension;

D_2 –correlation exponent.

They are connected by the rigorous inequalities:

$$D_0 \geq D_1 \geq D_2. \tag{4}$$

There are examples showing that these dimensions diverge considerably reflecting the inhomogeneity of the attractor [17]. The computation of D_q from time series is based on a reconstructed m-dimensional pseudo-phase space [18], for which a time lag τ is needed. Our results are not significantly influenced by the choice of τ (tested for $\tau = 1,\ldots,12$). The N points in the reconstructed phase space are denoted by x_j and the Euclidean norm is used. A local density function n_j is defined by

$$n_j(r) = \frac{1}{N-1} \sum_{j \neq k} \theta(r - x_j - x_k), \tag{5}$$

$$\theta(s) = \begin{cases} 1, & \text{if } s \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

The D_q values are estimated by different ways of averaging (cf. [12, 17, 19])

$$C_q(r) = \left(\frac{1}{N} \sum_{j=1}^{N} \left(n_j(r)\right)^{q-1} \right)^{1/(q-1)} \sim r^{\bar{D}_q}. \tag{6}$$

We obtain $\bar{D}_0$ and $\bar{D}_2$ with the harmonic ($q \to 0$) and the arithmetic ($q = 2$) mean, respectively. The estimated information dimension $\bar{D}_1$ is given by the geometric mean of n_j, the bars marking approximated values (cf. eq. (6)).

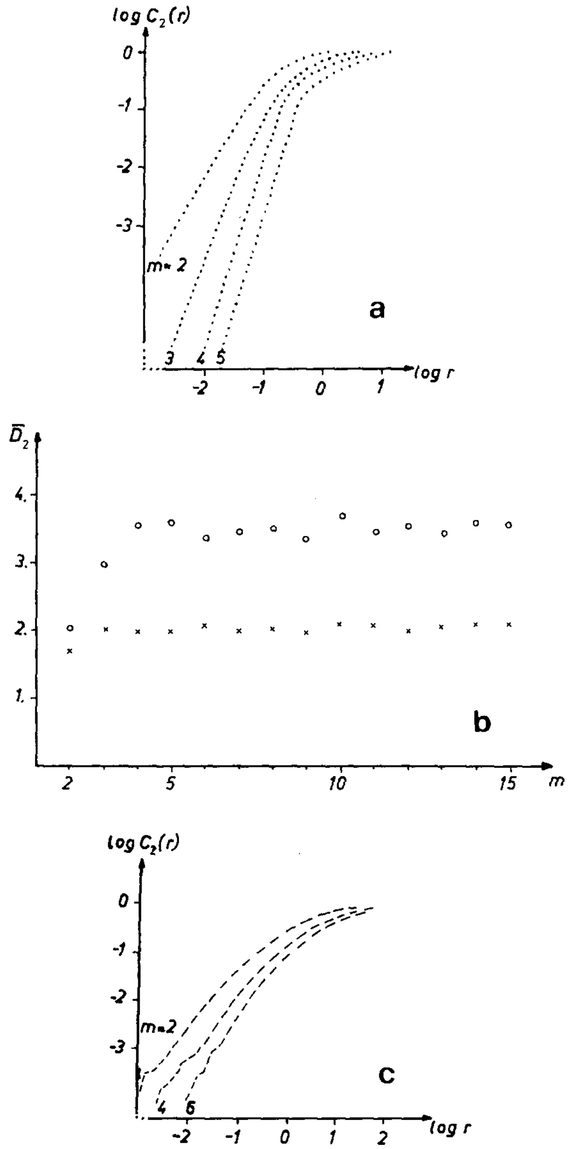


Fig. 4. a) Log C_2 versus log r of the solar measurement. b) Dependence of the $\bar{D}_2$ corresponding to (6) on the embedding dimension m. × – Lorenz model; ∘ – solar data. c) Log C_2 versus log r of the AR-5 process.

The log C_2 vs. log r plot from the solar record displays a good agreement with a power law for

Table I
Estimates of attractor dimensions

Data	Dimension	$\bar{D}_0$	$\bar{D}_1$	$\bar{D}_2$
Periodic signal	1	1.03 ± 0.05	1.03 ± 0.05	1.025 ± 0.05
Lorenz model	2.06 [12]	2.11 ± 0.1	2.11 ± 0.1	2.05 ± 0.1
Solar record	?	3.8 ± 0.3	3.7 ± 0.3	3.5 ± 0.3

intermediate r (fig. 4a). The dependence of the slope $\bar{D}_2(m)$ on the embedding dimension is drawn in fig. 4b. $\bar{D}_2$ reaches a saturation limit beyond $m = 4$. The results of our dimension measurements are summarized in table I. We can conclude:

1) The surprising agreement between the estimated and the known dimension of the models b and c supports the idea that the structure of the attractor can be analyzed even if the available data lengths are rather short (cf. [20, 21]).

2) There is a low-dimensional attractor underlying the observed solar pulsations.

3) The differences between the various dimensions suggest that the attractor is relatively inhomogeneous.

Returning to the AR-5 model, we notice that the log vs. log plot in fig. 4c is entirely different to fig. 4a. A linear increase is missing in fig. 4c, this again indicating the structural differences between the solar record and the stochastic AR-models.

5. Is the attractor chaotic?

The non-periodicity and the correlation decay are not necessarily caused by stochastic forces. They may be related to a deterministic chaos [2].

Besides the dimension we, therefore, discuss the stability of trajectories and the Kolmogorov entropy. A positive Lyapunov exponent reflects the orbital instability, and the Kolmogorov entropy measures how rapidly the information about the initial state region is lost as time increases [22].

5.1. *Maximum Lyapunov exponent*

Lyapunov exponents λ_i are defined by the behavior of infinitesimal deviations from the trajectory [2]. If we take at time $t = 0$ an infinitesimal hypersphere centered at an attractor point x_0 it is transformed into an ellipsoid with the semi-axes

$$b_i(t) \approx b_i(0) \exp(\lambda_i t). \tag{7}$$

Therefore, the growth of the distances of nearby points has to be studied [23]. First, we discuss a simple procedure which determines a lower limit $\bar{L}$ of the maximum Lyapunov exponent λ_{max}; we then apply the algorithm proposed by Wolf et al. [14].

1st method: We choose all pairs of nearby points in a pseudo-phase space satisfying

$$r_0^{(j,k)} = |x_j - x_k| < \varepsilon \tag{8}$$

and compute the distances of these points which belong to the trajectories after n steps,

$$r_n^{(j,k)} = |x_{j+n} - x_{k+n}|. \tag{9}$$

In this way, we obtain a stretching factor from any pair of adjacent points

$$d_n^{(j,k)} = \frac{r_n^{(j,k)}}{r_0^{(j,k)}}. \tag{10}$$

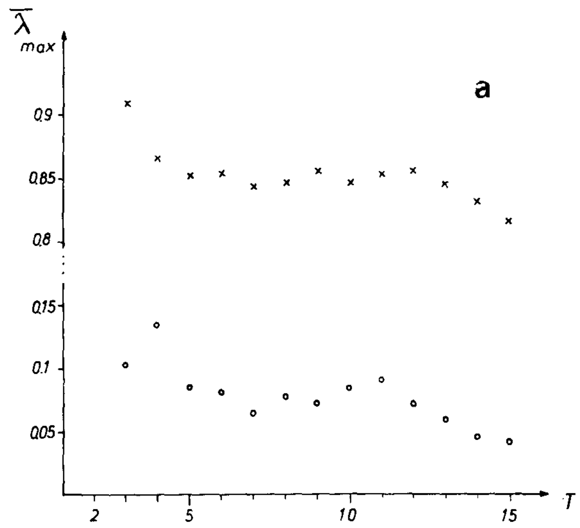
Averaging leads to

$$\bar{L}(n) = \frac{1}{N} \sum_{j,k} \ln d_n^{(j,k)}. \tag{11}$$

For small n the $\bar{L}(n)$ oscillates (reflecting the inhomogeneous growth of the distances [17]). Otherwise $r_n^{(j,k)}$ should be smaller than the attractor size. In view of these restrictions we have chosen $n = 8$ in table II. This method necessarily leads to an underestimation of λ_{max} due to the fact that not only the direction of maximal expansion is taken into account. However, a positive $\bar{L}$

Table II
Estimates of the maximum Lyapunov exponent

Data	λ_{max}	$\bar{L}$	$\bar{\lambda}_{max}$
Periodic signal	0	−0.002	0.0003
Lorenz model	0.91 [12]	0.75	0.85
Solar record	?	0.01	0.08



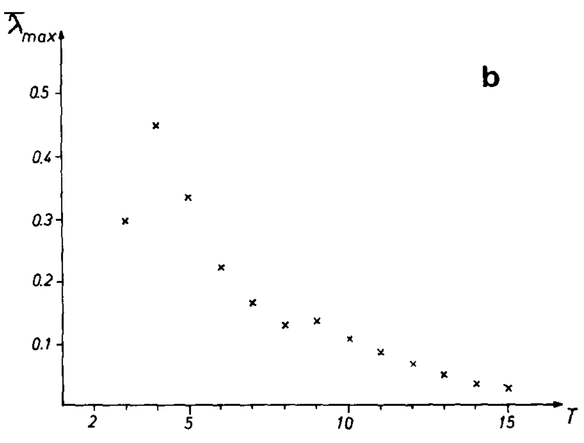


Fig. 5. a) Dependence of the estimated maximum Lyapunov exponent on the evolution length T. × – Lorenz model; ∘ – solar record. b) Same as fig. 5a, but for the AR-5 process.

(see table II) implies a positive Lyapunov exponent.

2nd method: The algorithm proposed by Wolf [13, 14] includes the search for the direction of maximum expansion. The long-term evolution of the separation of nearby points is investigated. Always after an evolution time T the second point of the pair is replaced by a "nearest neighbor" of the first, subject to the condition that the orientation of the separation vector is most nearly preserved. The average rate of the growth of the logarithm of this separation is the λ_{max} estimate in table II. In fig. 5a the dependence of $\bar{\lambda}_{max}$ on the evolution time T is presented for the Lorenz model and the solar pulsation event. We notice in both cases an exponential growth over several scales. A comparison of the corresponding plot of the AR-5 model, where no plateau can be observed (fig. 5b), is rather instructive. We have found equivalent results in a study of white noise. Therefore, we conclude that the $\bar{\lambda}_{max}$ vs. T plot offers a way to distinguish between stochastic separation (diffusion) and chaos [17].

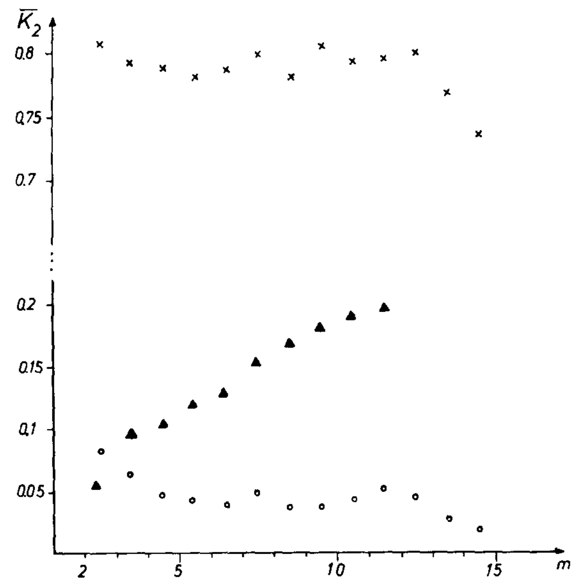


Fig. 6. Values of K_2 for different m. × – Lorenz model; ∘ – solar record; ▲ – AR-5 process.

5.2. *Kolmogorov entropy*

Due to the exponential divergence of nearby trajectories information is produced by the system. The mean rate of creation of information is measured by the Kolmogorov entropy K (cf. [21, 24]). Grassberger and Procaccia have proposed an algorithm leading to a lower limit K_2 of K [11], which is obtained from the vertical spacing between the parallel lines of the log C_2 vs. log r plot in fig. 4a. $\bar{K}_2$ is presented in fig. 6 for various values of the embedding dimension m. We have obtained plateaus for the Lorenz model and the solar record but not for the AR-5 model (where $\bar{K}_2$ increases with increasing m),

Lorenz model: $\bar{K}_2 = 0.79$;
solar record: $\bar{K}_2 = 0.04$.

6. Conclusions

In the present paper we have analyzed a record of a solar pulsation event. The data are characterized by a broad-band power spectrum. There is insufficient evidence to prove the existence of periodic components expected from the usual models.

We have applied the concept of time series analysis concerning the properties of a possible attractor structure of the underlying dynamical system.

Due to the short duration of such pulsation events in the solar atmosphere, only a rather short record is available. The applicability of the algorithms to the estimation of dimensions, the maximum Lyapunov exponent and the Kolmogorov entropy may be questioned with such small data samples. Therefore, we have carried out calculations with a periodic signal and the Lorenz model using the same number of data. The accuracy of the resulting estimates is rather surprising, and the qualitative differences from a stochastic process are obvious. Such an AR-5 process has been constructed with a quite similar power spectrum as the solar record. Calculations for the Rossler-hyperchaos system [14], which has a substantially higher dimensionality compared to the Lorenz model, have provided similar accuracy of the estimates.

The estimated dimensions of the solar time series suggest that there exists a low-dimensional attractor. A positive Lyapunov exponent as well as the positive Kolmogorov entropy estimated from the data give some indications of deterministic chaos.

The quantities describing deterministic chaos are interrelated. Particularly, the rigorous inequalities

$$K_2 \leq K \leq \Sigma^+ \lambda_i \tag{13}$$

are valid [2] (Σ^+ denotes the sum over the positive Lyapunov exponents). Furthermore, the decay of correlations is often connected with λ_{max} [25]. Hence, our approximations are self-consistent since $\bar{\lambda}_{max}$, $\bar{K}_2$ and the inverse correlation time of about 0.1 are of the same order of magnitude.

The properties of the attractor deduced from the data cannot be explained by a superposition of periodic components or by a stochastic AR model. Therefore, the application of linearized physical models is not admissible. The observed phenomenon should be described by non-linear systems which are capable of deterministic chaos. Approximations using only some modes could work successfully due to the low attractor dimensions [2].

Finally, we should like to emphasize that the existence of a chaotic attractor is not a well-established fact despite the above indications. The description of solar records under this point of view is still in its infancy and should be continued by modelling the physical processes and further analyses of time series.

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