Simulation of large scale cortical networks by use of individual neuron dynamics

G. Schmidt[1], G. Zamora-López[1] and J. Kurths[2,3]

[1]Interdisciplinary Center for Dynamics of Complex Systems,
University of Potsdam, 14469 Potsdam, Germany.
[2]Potsdam Institute for Climate Impact Research,
Telegrafenberg A 31, Postfach 60 12 03, 14412 Potsdam
[3]Humboldt University, Institute of Physics,
Newtonstr. 15,12489 Berlin

October 17, 2008

Abstract

Understanding the functional dynamics of the mammalian brain is one of the central aims of modern neuroscience. Mathematical modelling and computational simulations of neural networks can help in this quest. In recent publications, a multilevel model has been presented to simulate the resting-state *dynamics of the cortico-cortical connectivity of the mammalian brain. In the present work we investigate how much of the dynamical behaviour of the multilevel model can be reproduced by a strongly simplified model. We find that replacing each cortical area by a single Rulkov map recreates the patterns of dynamical correlations of the multilevel model, while the outcome of other models and setups mainly depends on the local network properties, e.g. the input degree of each vertex. In general, we find that a simple simulation whose dynamics depends on the global topology of the whole* *network is far from trivial. A systematic analysis of different dynamical models and coupling setups is required.*

1 Introduction

Understanding the functional dynamics of the mammalian brain is one of the central aims of modern neuroscience. Apart from developing and improving experimental methods like EEG, fMRI and PET, mathematical modelling and computational simulations of neural networks can help to understand the dynamical behaviour of the brain. While the dynamics of single neurones is quite well conceived and their dynamical behaviour can be reproduced by differential models [2, 10] e.g. Hodgkin-Huxley [5] or iterated maps, e.g. Rulkov [14], a systematic exploration of the impact of connection topology on the dynamical organisation in neuronal networks is barely beginning [6, 11, 13, 22, 23].

In [2, 24] the authors simulated the resting-state of the mammalian cortex using different approaches. The models were implemented using the experimentally known anatomical connectivity of the cat cortex, which consists of a network of 53 cortical areas and 826 long-range fibers connecting them [15, 16]. The first approach was to simulate cortical areas using a neural mass model [19], which aims to imitate the dynamics of a population of neurones and is capable of reproducing EEG-like oscillations. Hence, it promises to be a biologically plausible model for a brain area. However, it was found that the pairwise correlation between cortical areas depends on the individual characteristics of the areas (number of inputs) but further properties of the network topology were irrelevant. The second approach comprised of a multilevel model (a network of networks) in which each area was represented by a subnetwork of 200 neurones. While the connectivity between the cortical areas remained the same, the internal 200 neurones were connected by a small-world topology [18] in order to minimally reproduce qualitative observations of neuronal connectivity [17]. This model displayed biologically plausible behaviour in the range of weak coupling between neurones, meaning that (partial) synchronisation and dynamical clustering followed the underlying anatomical topology. Additionally, the mean field signals of the areas showed similarity to real EEG data. In the strong coupling regime, the dynamics of individual areas were characterised by well-defined oscillations (as happened with the neural mass model) and therefore, the patterns of synchronisation did not depend on the underlying topology. One of the drawbacks of the multilevel model is its rather large computational cost.

The goal of the present work is to investigate how much of the dynamical behaviour of the multilevel model can be reproduced by a strongly simplified model (which is computationally competitive). For that, we have simulated each cortical area as a single neuron. Using two popular neuron models, the Rulkov map [14] and the neuron model by Izhikevich [8], the network is simulated and their outcomes are compared. We find that the network of interconnected Rulkov maps recreates the patterns of dynamical correlations of the multilevel model, while in the simulations performed using the Izhikevich model, the dynamics does not depend on the underlying topology. Among other factors, which are discussed below, the difference might arise from the fact that the Rulkov maps are connected by a linear coupling and the Izhikevich neurones by a pulse coupling.

Nevertheless, we find that in the simulations using the Rulkov map, the neurones have a flexible dynamics permitting them to quickly adapt to the dynamics of their neighbours. In the simulations performed with the Izhikevich neurones, each element follows mainly its own dynamics and correlates only with the neighbours of strongest input. This observation transcends the particular models and configurations here adopted and should be taken on account when studying synchronization phenoma in complex networks [1, 3].

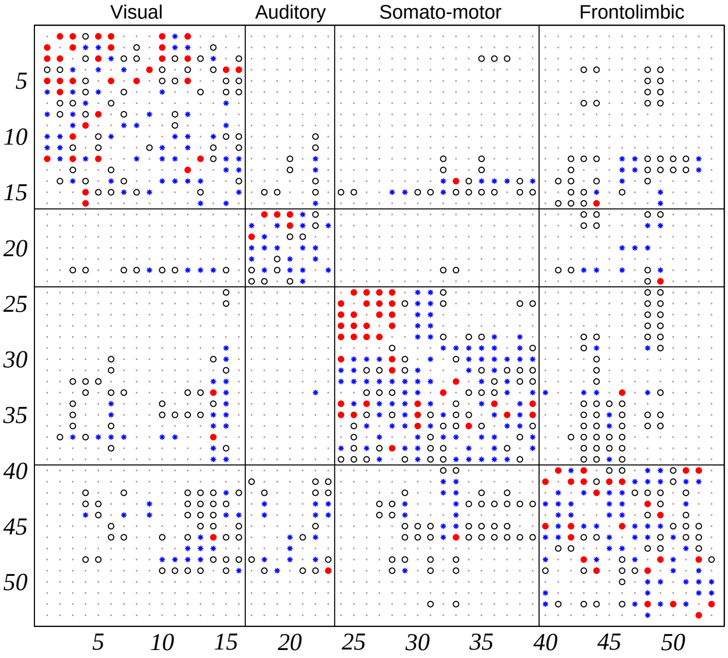


Figure 1: Weighted adjacency matrix $\boldsymbol{W}$ of the cortico-cortical connectivity of the cat comprising of 826 directed connections between 53 cortical areas. The connections are classified as weak (open circles), intermediate (blue stars) and dense (red filled circles) according to the axonal densities in the projections between two areas. For visualisation purpouses, the non-existing connections (0) have been replaced by dots.

2 Dynamical Simulations of the Cat Cortex

As in [2, 23, 24] the cortico-cortical connectivity of the cat has modelled, which is summarised as a weighted adjacency matrix $\boldsymbol{W}$, Fig. 1. This data is a collation performed by J. W. Scannell [15, 16] based on previous anatomical reports. The network is weighted according to the axonal density of the fibers linking two areas. Thus, 3 means a strong (dense) connection, 2 means intermediate and 1 a weak (sparse) connection. The cases explicitly reported as *absent* or *no information available* were treated the same and were weighted as 0. Cortical networks have been reported to have small-world characteristics, i.e. a high clustering coefficient and a short average pathlength [4]. The degree of the nodes is very heterogeneous and ranges from 2 to 35 connections, insofar, the network has a (scale-free-like) broad degree-distribution [12, 20]. In Figure 1, four main clusters, called "communities", can be distinguished which have dense connectivity inside. These communities correspond to the functional subsystems *visual (V), auditory (A), somato-motor (SM)* and *frontolimbic (FL)* [4, 15, 16].

In order to achieve our goal of developing a model that reproduces the main dynamical behaviour of the multilevel model and is computationally competitive, we choose to turn back from the multilevel approach and try alternative dynamics for modelling the areas. The dynamics of an area is now simulated by a single neuron, modelled by 1) an iterated map developed by N. F. Rulkov [14], and 2) a system of two first order differential equations (ODEs) with an additional after spike reseting mechanism, after E. Izhikevich [8]. Every neuron receives input from its neighbours and is additionally stimulated by an independent Gaussian white noise.

2.1 Simulation of Cortical Areas by the Rulkov Map

The Rulkov map is a two-dimensional iterated map, which is capable of replicating the spiking, and spike-bursting behaviour of real neurones by the interplay between two dynamical variables, one describing fast dynamics and the other slow variable (see details in Appendix A). To simulate the cortical network of the cat, each area is replaced by one Rulkov map in *chattering* state, i.e. the isolated neuron is characterised by periodic epochs of bursting and inactivity, see Fig. 2 (Top). The maps were then coupled following the recommendations in [14]. After being coupled, the maps still display a chattering behaviour but irregular due to the input from their neighbours, Fig. 2 (Bottom).

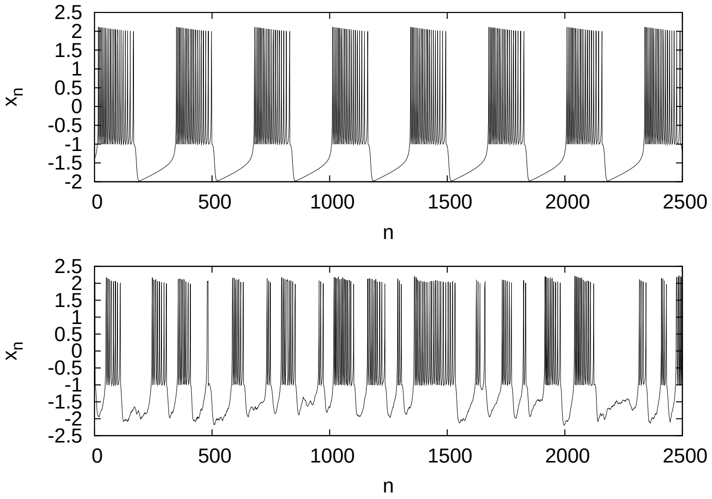


Figure 2: Characteristic signal of a cortical area modelled by a Rulkov map. (Top) Before coupling, isolated neurones were setup into a regular *chattering* state, i.e. periodic epochs of bursts and inactivity without noise. (Bottom) After the neuron is coupled to the network with noise, it displays an *irregular* chattering behaviour.

After running the simulations, the pairwise linear correlation $r(i,j)$ between cortical areas have been computed to characterise the synchronisation behaviour and the functional clustering. As mentioned in [7, 14], synchronisation is only observed in the low frequency bands of bursts, but not in high frequent spikes, so a low pass filter has been applied to the output signals before computing $r(i,j)$ (Appendix C). In Fig. 3 a gallery of correlation matrices $r(i,j)$ is shown and the corresponding cluster trees (dendrograms) for coupling strengths ranging from $g = 10$ to $g = 525$. Each correlation matrix is the average of 10 realisations. With a very weak coupling, Fig. 3(a), the dynamical clusters are visible although the correlations are weak and noisy. The dynamical clusters in Fig. 3(b) raise from the background with high contrast, while in Fig. 3(c) inter cluster correlation becomes more dominant. Finally, a strong coupling leads to global correlation, Fig. 3(c).

Next, a cluster analysis of the correlation matrices is performed by calculating their corresponding dendrograms. A dendrogram is a graph of many U-shaped lines, which connects nodes in a hierarchical binary tree. The height of each line stands for the *dynamical distance* (euclidean) between two nodes, which is obtained from the correlation matrix: $d_{ij} = \sqrt{\sum_m (r_{i,m} - r_{j,m})^2}$. This technique enables one to visualise which areas are closely connected and how they build up dynamical clusters. As we are concerned about the anatomical clusters (V, A, SM and FL), we concentrate on the hierarchical level where the correlation matrix decomposes into four clusters. Apparently, the stability of hierarchical levels is a function of correlation. By stability, we mean the height of the hierarchical level where there are four clusters in the dendrogram.

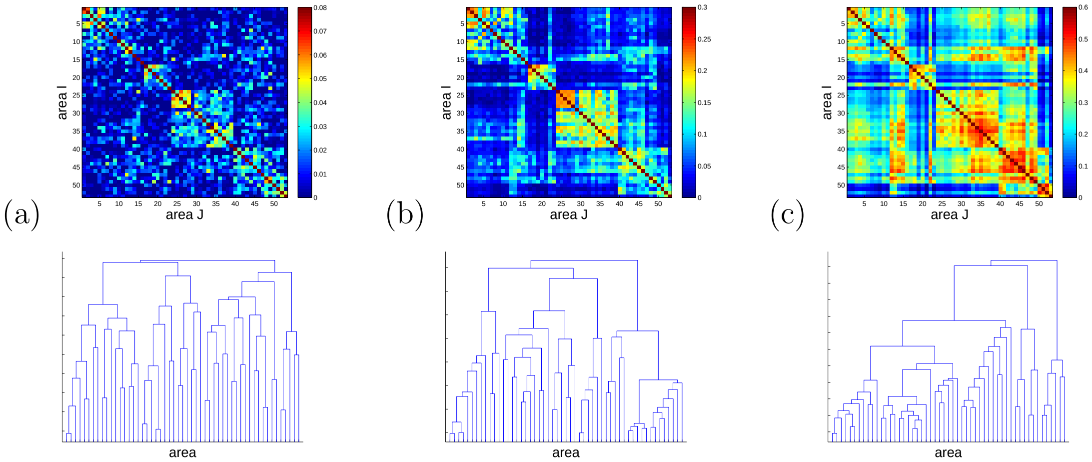

(a) (b) (c)

Figure 3: Correlation matrices and dendrograms for the Rulkov map with different coupling strengths. (a) $g = 10$, (b) $g = 75$ and (c) $g = 525$.

We find that, with low correlation, Fig. 3(a), the stability of our region of interest is very weak. Here, the correlation between single nodes is dominating the synchronisation behaviour, while an intermediate correlation, Fig. 3(b), leads to the desired synchronisation of four stable clusters where each cluster has its particular dynamics and is weakly coupled to other clusters. Very high coupling strength destabilises the four clusters and "melt" them together, which means that individual dynamics is getting lost, while synchronisation of all clusters becomes dominant, Figs. 3(c). Note, that in these dendrograms, areas are generally not ordered by their number. To see which areas form a cluster, we provide another graph (Fig. 4). The dynamical clusters closely follow the anatomical communities, although some of the areas are assigned to a different community. These areas, while belonging to an anatomical community, are hubs with many connections in other communities. Therefore, they also get synchronised with other communities and appear dynamically clustered into a "wrong" community. The presence of such hubs has also been reported in [6, 21, 22, 23, 24].

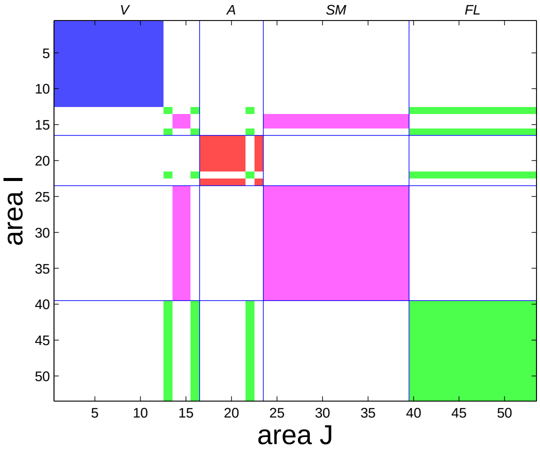


Figure 4: The four dynamical clusters of the Rulkov model at an intermediate coupling strength of 75, result of the clustering algorithm described in Section 2

2.2 Simulation of Areas by Izhikevich Neurones

The same analysis has been repeated using the Izhikevich neuron to model the cortical areas [8]. This model consists of a pair of first order ODEs; one variable is denoted as the *membrane potential of the neuron* and the other as *a membrane recovery variable*. Additionally, it contains an after-spike reset to avoid modelling the shape of the spike. This model has been conceived to reproduce a rich range of dynamical behaviours exhibited by real neurones of different classes in a computationally competitive manner [8, 9].

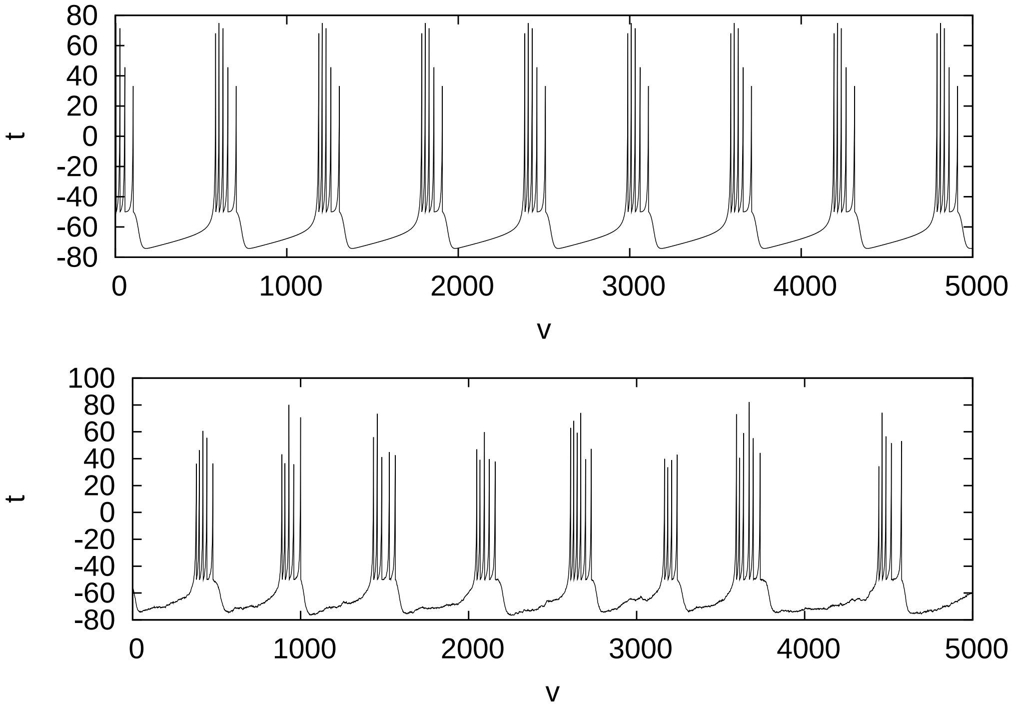


Figure 5: Characteristic signal of a cortical area modelled by the Izhikevich neuron. (Top) Before coupling, isolated neurones were setup into a regular *chattering* state without noise. (Bottom) After the neuron is coupled into the network with noise, it still displays a chattering behaviour, although slightly irregular.

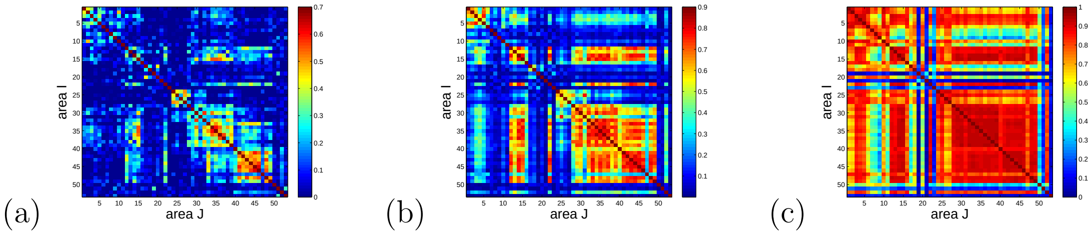


Figure 6: Correlation matrices for the Izhikevich map with different coupling strengths. (a) $g = 3$, (b) $g = 5$, (c) $g = 10$, at a noise level of $D = 0.1$

The model parameters were chosen to simulate the isolated neurones in the *chattering* state as shown in Fig. 5 (Top). The network was implemented, setting up the neuron to neuron pulse-coupling as indicated in [8] (see details in Appendix B). In the coupled configuration, areas still display a chattering behaviour, although only slightly irregular, Fig. 5 (Bottom), as compared to the coupled Rulkov maps, Fig. 2 (Bottom).

The simulated network also yields synchronisation, but as seen in Fig. 6, the clustering behaviour is different. Clusters are not concordant with the anatomical communities, regardless of the coupling strength. Here, the SM and FL communities follow almost the same dynamics, together with a large part of the visual cluster. Moreover, the auditory community disappears as a dynamical cluster. These observations are very similar to the patterns of synchronisation of the neural mass model discussed in [24].

3 Summary and Discussion

In this paper we have explored an alternative and simplified manner to simulate cortical networks which can still recover the dynamical properties of the multilevel approach of [2, 21, 22, 23, 24]. We have simulated the cortico-cortical network of the cat by replacing each cortical area by a single excitable oscillator: either a

Rulkov map or an Izhikevich neuron. The dynamical correlations between areas, using the Rulkov map (Fig. 3) closely resembles those of the multilevel model. In the weak coupling regime, both models display dynamics which depend on the underlying topology. The dynamical clusters are similar to the anatomical communities (V, A, SM and FL). An example of the dynamical clusters obtained with the multilevel model are shown in Fig. 7 for comparison. In the strong coupling regime of both models, the dynamical clusters melt together leading to global synchronisation, a situation which might be considered as *pathological* in brain activity - e.g. during epileptic seizures. On the contrary, in none of the simulations performed using the Izhikevich neurones, we could reproduce this dynamical clustered behaviour (see Fig. 6).

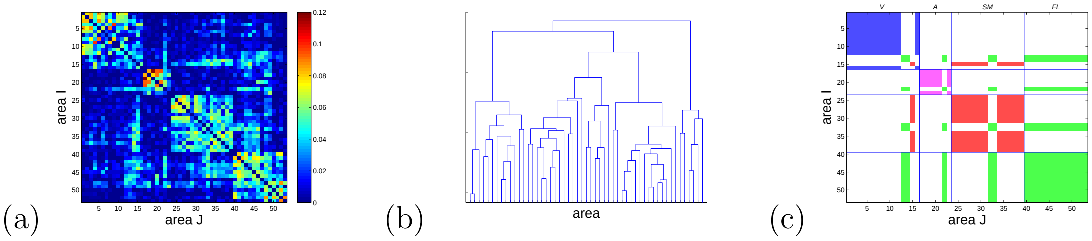


Figure 7: Multilevel model of [2, 23, 24] (a) correlation matrix, (b) dendrogram and (c) clustering map in a weak coupling regime.

An answer to this difference may lie in the fact that low dimensional maps are capable of rich dynamics (even 1-D maps can have chaotic behaviour), while differential equations can show chaotic behaviour only when they are at least 3-dimensional. The neurones modelled by the Rulkov map have flexible dynamics as shown in Fig. 2(b). As a consequence, the neurones are able to quickly adapt to the dynamics of their neighbours. For example, the hubs are connected to many areas which might have different time-scales. This adaptability permits the hubs to "keep in synch" with all their neighbours. In the simulations performed with the Izhikevich model, the individual bursting dynamics of the neurones is dominant and they essentially keep their own rhythm. As a result, areas can only synchronise with the groups of neighbours from which they receive strongest synchronised input.

Another reason for the different behaviours observed lies in the coupling between neurones. In the Rulkov model the neurones are connected by a linear coupling which leads to burst synchronisation. As recommended in [8] the connections in the Izhikevich model are modelled by a pulse coupling between the fast variables of both neurones. Hence, the input of a neuron takes only into account the number of neighbours which fire a spike simultaneously.

Whether the dynamical flexibility of the areas arises from the chaoticity of the neuronal model and/or the type of coupling, it seems that when simulating a network of coupled oscillators (generic or excitable) the dynamics of the individual vertices need to be adaptable. Otherwise, the dynamical outcome is not sensitive to the underlying topology of the network, or trivially dependent on the local number of neighbours. Further preliminary observations with different setups, e.g. Izhikevich neurones with electrical coupling (not shown), exhibit dramatic changes in the correlation patterns. These observations suggest that a systematic analysis of different model dynamics and coupling types is required to understand when the outcome of a network simulation will depend on the underlying network topology or not.

Finally, we discuss the reliability of the simplified models presented here in order to simulate cortico-cortical networks. In the multilevel model, the time series signal of individual cortical areas, computed as the mean-field signal of the internal 200 neurones resemble typical EEG data, and thus it can be considered as a plausible model for a direct comparison to experimental-like data. In the simplified models the signals of individual areas is a spiking or a spike bursting pattern, hence interpreting it as EEG data is implausible. Nevertheless, if we regard the low pass filtered output as the signal (Appendix C), it might still have some similarities with experimental EEGs. In any case, as the simplified model captures the underlying topology of the network and computes very fast, it seems suitable for applications to investigate network synchronisation and other dynamics related properties like information processing. Another promising alternative is to build a multilevel model but using Rulkov maps instead of neuronal models based on differential equations.

As brain areas consist of a large number of neurones, which altogether have a very high-dimensional phase space, in which chaotic behaviour is likely, iterated maps could be interpreted as Poincaré-map of such a high dimensional system and hence, are more suitable to represent them as two-dimensional ODEs.

A The Rulkov Map

Rulkov introduced a model [14] that replicates the spiking and spike-bursting behaviour of real neurones. The model is represented by a two dimensional map:

$$
\begin{aligned}
x_{i,n+1} &= f(x_i, y_i + \beta) + \xi_i && (1)\\
y_{i,n+1} &= y_{i,n} - \mu(x_{i,n+1}) + \mu\sigma C(x_i, x_j) + \xi_i && (2)
\end{aligned}
$$

$$
\begin{aligned}
f(x_i, y_i) &= \begin{cases} \alpha/(1-x_i) + y_i, & x_i \leqslant 0 \\ \alpha + y_i, & 0 \leqslant x_i \leqslant \alpha + y_i \\ -1 & x_i \geqslant \alpha + y_i \end{cases} && (3)\\
C(x_i, x_j) &= \frac{g}{N}\sum_{j=1}^{N} \widetilde{W}_{ij}(x_i - x_j). && (4)
\end{aligned}
$$

constant	value
α	6
β	1
μ	0.001
σ	0.3

Table 1: Parameter values of the Rulkov map used for the simulations in Sec. 2.

It consists of a fast variable (x), which represents the membrane potential of the neuron and a slow one (y), enabling the possibility to operate on different time scales. For the fast dynamics of x, one can regard y as a control parameter. For some regions of y there exists a stable cycle in the fast dynamics of x, which generates periodic spiking. As y changes slowly, the system will undergo a bifurcation that destroys the stable cycle in the fast dynamics and x will converge to a fixed point, the relaxation regime. After some time y returns to the regime, where a stable cycle in x exists and everything starts over again.

Note that the adjacency matrix $\widetilde{W_{ij}}$ is the normalised weighted matrix after dividing the weight matrix by its largest element i.e. $\widetilde{W_{ij}} = W_{ij}/3$. The constants α, β, σ, μ are model parameters (Table 1) and are all the same for every area, while $\xi = D\Xi_{0,1}(t)$ stands for Gaussian distributed white noise with zero average and variance 1 multiplied by a constant (D) to adjust the noise level. The coupling strength is denoted as g.

B The Izhikevich Model

The model by E. M. Izhikevich [8] consists of a pair of first order ordinary differential equations:

$$\frac{d}{dt}v_i = 0.04v_i^2 + 5v_i + 140 - u_i + I_i \tag{5}$$

$$\frac{d}{dt}u_i = a(bv_i - u_i) + \xi_i \tag{6}$$

with an after-spike resetting

$$\text{if } v \geqslant 30, \text{ then } v := c \text{ and } u := u + d. \tag{7}$$

In our simulations, the equations have been integrated, using the fourth-order Runge-Kutta method with a step size of $dt = 0.1$[1]. This model is also capable of generating spiking and spike-bursting signals, but, as it consists of a system of two first order ODEs, it cannot be chaotic in the uncoupled case. Following the original recomendations, the pulse-coupling between neurones is modelled by replacing the input to the neurones I with:

$$I_i = I_0 \left(1 + \frac{g}{N}\sum_{j=1}^{N} \widetilde{W}_{ij}\Theta(v_j - 20)\right) + \xi_i \tag{8}$$

where $\widetilde{W}_{ij}$ is the adjacency matrix as above, g is the coupling parameter and $\xi = D\Xi_{0,1}(t)$ stands for Gaussian distributed white noise with zero average and variance 1 multiplied with a constant (D) to adjust the noise level. Here $D = 0.1$ was used in all cases. The control parameters (Table 2) were chosen such that the model generates a "chattering" signal $v(t)$, as shown in Fig. 5.

---

[1] For a faster computation, the simple Euler integration method can be used, because its errors do not accumulate due to the after-spike reset.

constant	value
a	0.02
b	0.2
c	-50.0
d	2
I_0	10

Table 2: Parameter values for the Izhikevich ODE.

C Pre-Processing

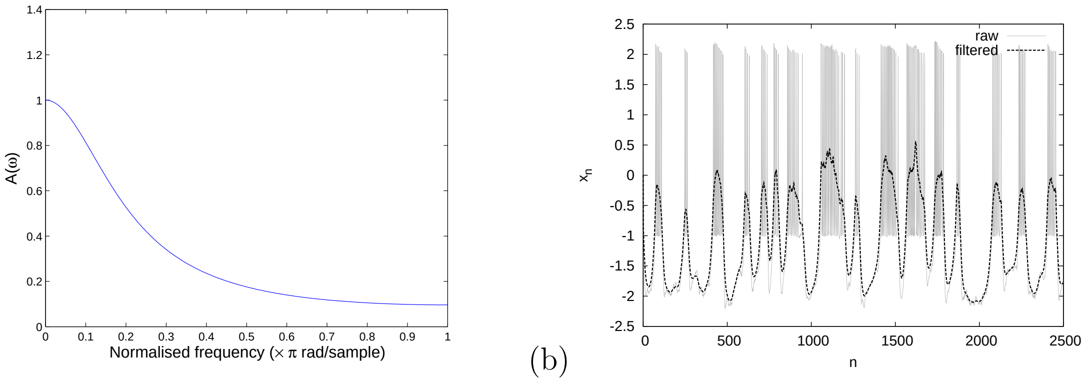
All simulation runs started with random initial conditions and the first, transient 10,000 iterations were discarded. The following 50,000 iterations were saved for data analysis. The correlation matrices exhibited are an average of 10 realisations for the Rulkov model and 50 for the cases using the Izhikevich model because of the resolution differences of both models. As a measure for synchronicity of the signals $x_{i,n}$ the linear (zero-lag) correlation coefficient $r(i,j)$ is used. The signal contains two main frequencies: a (fast) occurrence of spikes and a (slow) occurrence of bursts. Interested in the correlation of bursting activity, the signals have been filtered to eliminate the high frequencies. A recursive filter was chosen:

$$z_n = (a-1)x_n + a z_{n-1} \tag{9}$$

The filter has applied in both directions (forward and backward) to remove the phase shift that it generates if applied only once. We found $a = 0.9$ as suitable value. Fig. 8 shows the effect of the filter on our data.

Acknowledgments

We thank André Bergner and Jonathan Dongues for constructive discussions. G. Z.-L. and J. K. are supported by Deutsche Forschungsgemeinschaft (grants



(a) (b)

Figure 8: (a) Filter response, (b) Comparison between the filtered and the unfiltered (Rulkov) signal

EN 471/2-1, KL 955/6-1, and KL 955/14-1).

References

[1] A. Arenas, A. Díaz-Guilera, J. Kurths, Y Moreno, and C. S. Zhou. Synchronization in complex networks. Physics Reports, *in press*, 2008.

[2] P. beim Graben, C. S. Zhou, M. Thiel, and J. Kurths, editors. *Lectures in Supercomputational Neuroscience.* Springer-Verlag, Berlin, 2008.

[3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. *Phys. Reps.*, 424:175–308, 2006.

[4] C. C. Hilgetag, G. A. P. C. Burns, M. A. O'neill, J. W. Scannell, and M. P. Young. Anatomical connectivity defines the organization of clusters of cortical areas in the macaque monkey and the cat. *Phil. Trans. R. Soc. Lond. B*, 355:91–110, 2000.

[5] A. L. Hodgkin and A. F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol.*, 117:500–544, 1952.

[6] C. J. Honey, R. Kötter, M. Breakspear, and O. Sporns. Network structure of cerebral cortex shapes functional connectivity on multiple time scales. *Proc. Nat. Acad. Sci.*, 104:10240–10245, 2007.

[7] M. V. Ivanchenko, G. V. Osipov, V. D. Shalfeev, and J. Kurths. Phase synchronization in ensembles of bursting oscillators. *Phys. Rev. Lett.*, 93:134101, 2004.

[8] E. M. Izhikevich. Simple model of spiking neurons. *IEEE Trans. Neural Nets.*, 14:1569–1572, 2003.

[9] E. M. Izhikevich. Which model to use for cortical spiking neurons? *IEEE Trans. Neural Nets.*, 15:1063–1070, 2004.

[10] E. M. Izhikevich. *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting.* MIT Press, Cambridge, MA, 2007.

[11] E. M. Izhikevich and G. M. Edelman. Large-scale model of mammalian thalamocortical systems. *Proc. Nat. Acad. Sci.*, 105(9):3593, 2008.

[12] M. Kaiser, R. Martin, P. Andras, and M. P. Young. Simulation of robustness against lesions of cortical networks. *Eur. J. Neurosc.*, 25:3185–3192, 2007.

[13] H. Markram. The blue brain project. *Nat. Rev. Neurosci.*, 7:153—-160, 2006.

[14] N. F. Rulkov. Modeling of spiking-bursting neural behavior using two-dimensional map. *Phys. Rev. E*, 65:041922, 2002.

[15] J. W. Scannell, C. Blakemore, and M. P. Young. Analysis of connectivity in the cat cerebral cortex. *J. Neurosci.*, 15(2):1463–1483, 1995.

[16] J. W. Scannell and M. P. Young. The connectional organization of neural systems in the cat cerebral cortex. *Curr. Biol.*, 3(4):191–200, 1993.

[17] O. Sporns, G. Tononi, and R. Kötter. The human connectome: a structural description of the human brain. *PLoS Comput. Biol.*, 1(4):0245–0251, 2005.

[18] D. J. Watts and S. H. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440, 1998.

[19] F. Wendling, J. J. Bellanger, F. Bartolomei, and P. Chauvel. Relevance of nonlinear lumped-parameter models in the analysis of depth-eeg epiletic signals. *Biol. Cybern.*, 83:367–78, 2000.

[20] G. Zamora-López, C. S. Zhou, and J. Kurths. *Lectures in Supercomputational Neuroscience*, chapter Structural Characterization of networks using the cat cortex as an example. Springer-Verlag, Berlin, 2008.

[21] L. Zemanová, G. Zamora-López, C. S. Zhou, and J. Kurths. Complex brain networks: From topological communities to clustered dynamics. *PRAMANA J. Phys.*, 70(6):1087–1097, 2008.

[22] L. Zemanová, C. S. Zhou, and J. Kurths. Structural and functional clusters of complex brain networks. *Physica D*, 224:202–212, 2006.

[23] C. S. Zhou and *et al.* Hierarchical organization unveiled by functional connectivity in complex brain networks. *Phys. Rev. Lett.*, 97:238103, 2006.

[24] C. S. Zhou and *et al.* Structure-function relationship in complex brain networks expressed by hierarchical synchronisation. *New J. Phys.*, 9:178, 2007.