



Correlated noise induced spatiotemporal coherence resonance in a square lattice network

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ABSTRACT

The effects of additive correlated noise, which is composed of common Gaussian white noise and local Gaussian colored noise, on a square lattice network locally modelled by the Rulkov map are studied. We focus on the ability of noise to induce pattern formation in a resonant manner. It is shown that local Gaussian colored noise is able to induce pattern formation, which is more coherent at some noise intensity or correlation time, so it is able to induce spatiotemporal coherence resonance in the network. When common Gaussian white noise is applied in addition, it is seen that the correlated noise can induce coherent spatial structures at some intermediate noise correlation, while this is not the case for smaller and larger noise intensities. The mechanism of the observed spatiotemporal coherence resonance is discussed. It is also found that the correlation time of local colored noise has no evident effect on the optimal value of the noise strength for spatiotemporal coherence resonance in the network.

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1. Introduction

It is well-known that noise can play a constructive role in nonlinear dynamical systems. After the paper by Benzi, et al. [1], the phenomena of stochastic resonance (SR), that is, noise can enhance the response of a system to weak external periodic excitations, have attracted a lot of interest (see Ref. [2] and references therein). Another fascinating phenomenon is that noise can also play a constructive role even in the absence of external signals. This phenomenon was named as autonomous stochastic resonance [3] or coherence resonance (CR) [4], observed mainly in excitable systems [3–6]. In the past years, SR and CR have also been observed experimentally in diverse systems, such as excitable chemical reactions [7], the cat's neural system [8], or the human blood pressure regulatory system [9]. For a more detailed discussion of noise effects in excitable systems, we refer readers to [10]. Recently, the frontier of investigating the noise influences on nonlinear dynamical systems has shifted to spatially extended systems [11]. Inspired by the work of Carrillo et al. [12], Perc showed that additive or multiplicative spatiotemporal noise is able to extract a particular spatial periodicity of excitable media in a resonant manner [13,14]. The phenomena that noise is able to induce spatiotemporal coherence resonance are also reported in Refs. [15–17].

An important field of application of this approach is neuroscience [10]. Neurons in spatially extended systems are usually subjected to a large number of synaptic inputs from other neurons. These synaptic inputs exhibits random-like behavior and are spatially correlated among the neurons. The effects of this correlated noise on coherence of spatially extended systems have been discussed in Refs. [18–21]. In this paper, we will discuss the effect of additive noise, composed of common Gaussian white noise and local Gaussian colored noise, on a square lattice network which is locally modelled by the Rulkov map for active neurons [22,23]. The Rulkov model captures all the vital aspects of neuronal dynamics in a numerically very

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efficient way, which makes it an ideal candidate for studying noise-induced phenomena in extended systems, as exemplified recently in Refs. [24–26] for example. With the aid of a method for quantifying spatiotemporal patterns and detecting spatiotemporal stochastic resonance (STSR) as proposed by Hütt et al. [27], we show that the noise considered here is able to induce pattern formation and spatiotemporal coherence resonance (STCR).

The structure of this paper is arranged as follows. In Section 2, we introduce the equations of the square lattice network locally modelled by the Rulkov map. In Section 3, we study the effects of correlated noise on the spatiotemporal pattern formation of the studied system. Based on the results of Section 3, we utilize a linear spatial cross-correlation measure to quantify the pattern formation and exhibit the noise-induced spatiotemporal coherence resonance in Section 4. Finally, we summarize our results and draw some conclusions in Section 5.

2. Equations of the square lattice network

The equation of the Rulkov map is given as follows

$$\begin{cases} u_{n+1} = \alpha/(1 + u_n^2) + v_n \\ v_{n+1} = v_n - \beta u_n - \gamma \end{cases} \quad (1)$$

where u_n is the membrane potential of the neuron and v_n is the variation of the ion concentration. u_n, v_n represent the fast and slow dynamical variables, respectively. n is the iterated time index. α, β, γ are system parameters. Here β and γ are both taken as 0.001. In this case, it is found that for $\alpha < 2.0$, the map is governed by a single excitable steady state $(-1, -1 - \alpha/2)$, while for $\alpha > 2.0$ the excitable steady state loses its stability via a Hopf bifurcation. The Rulkov map is employed to model the behavior of neurons with self-sustained subthreshold oscillations. This paradigmatic model also replicates the dynamics of spiking-bursting activity of real biological neurons. The use of low-dimensional model maps is useful for understanding dynamical mechanisms in real neuron ensembles, since it can show typical restructuring of collective behavior and is simple enough to study the reason behind such restructuring. In this paper, we use the Rulkov map as the local unit in a square lattice network, which is described by the equations as follows:

$$\begin{cases} u_{n+1}(i, j) = \alpha/[1 + u_n^2(i, j)] + v_n(i, j) + D[u_n(i + 1, j) + u_n(i - 1, j) + u_n(i, j - 1) \\ + u_n(i, j + 1) - 4u_n(i, j)] + \eta(i, j)(n) \\ v_{n+1}(i, j) = v_n(i, j) - \beta u_n(i, j) - \gamma \end{cases} \quad (2)$$

where the subscript (i, j) indicates that the network unit locates on the i -th row and the j -th column, $i, j = 1, \dots, N$. We take $N = 128$ here. Each Rulkov map is coupled diffusively with its four nearest neighbors, and D is the coupling strength of the coupled system. The noise term is assumed to be $\eta(i, j)(n) = \sqrt{R}e(n) + \sqrt{1 - R}\xi(i, j)(n)$ with

$$\langle e(n) \rangle = 0, \langle e(n)e(n') \rangle = 2\sigma\delta(n - n') \quad (3)$$

and

$$\begin{cases} \langle \xi(i, j)(n) \rangle = 0 \\ \langle \xi(i, j)(n)\xi(i', j')(n') \rangle = \sigma\lambda\exp(-\lambda|n - n'|) \end{cases} \quad (4)$$

where $\langle \rangle$ indicates a statistic average. The Gaussian white noise $e(n)$ is common to all units, and $\xi(i, j)$ is the local Gaussian colored noise. The noise intensities of common noise and colored noise are taken as the same value σ , while λ^{-1} is the correlation time of the colored noise. The local noise term $\xi(i, j)$ is uncorrelated from site to site, namely, $\langle \xi(i, j)(n)\xi(k, l)(n) \rangle = 0$. Since

$$\langle \eta(i, j)(n)\eta(i', j')(n') \rangle = 2R\sigma\delta(n - n') + (1 - R)\sigma\lambda\exp(-\lambda|n - n'|)\delta_{i, i'}\delta_{j, j'} \quad (5)$$

the control parameter R measures the noise correlation between a pair of neurons. At each iterated step n for each unit, $\eta(i, j)(n) = \sqrt{R}e(n) + \sqrt{1 - R}\xi(i, j)(n)$ is replaced by a new random number, where $e(n)$ and $\xi(i, j)(n)$ are generated through the Box–Mueller algorithm and the integral algorithm presented in Ref. [28], respectively.

From Eq. (5), we can define the noise strength (that is, the total power) of the correlated noise $\eta(i, j)$ as $\sigma_\eta = \langle \eta(i, j)^2(n) \rangle = 2R\sigma + (1 - R)\sigma\lambda$. We can see that the noise strength of the correlated noise slides in the interval $[\sigma\lambda, 2\sigma]$ or $[2\sigma, \sigma\lambda]$ by changing R . In this paper, we set $\alpha = 1.99$, $D = 0.0025$, and take σ, λ, R as control parameters. For our simulations below, we consider periodic boundary conditions, namely, $u(0, j) = u(N, j)$, $u(N + 1, j) = u(1, j)$, $u(i, 0) = u(i, N)$, $u(i, N + 1) = u(i, 1)$, and take $(-1, -1 - \alpha/2)$ as initial conditions for all units. We will study the effects of correlated noise $\eta(i, j)$ on pattern formation and spatiotemporal CR of the network in the next two sections.

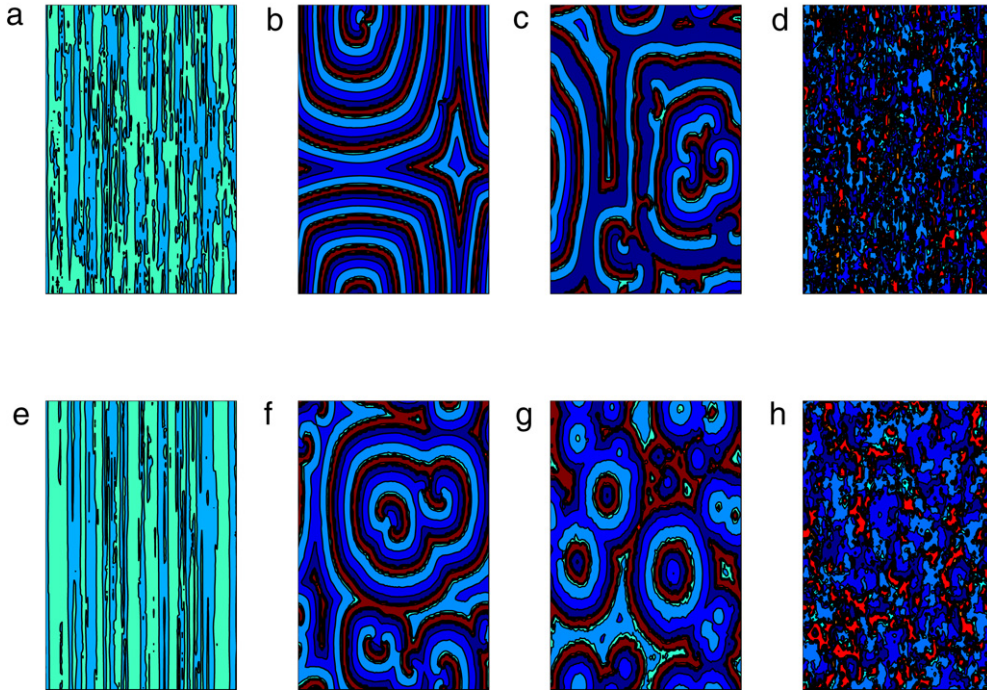


Fig. 1. Snapshots of $u(i, j)$ at grid points of the lattice with local Gaussian colored noise at $D = 0.0025$. All the snapshots of the spatial grid are taken at given times n . For the top row, fix $\lambda = 0.5$ and let σ_1 increase from left to right: $\sigma_1 = 0.0039, 0.004, 0.007, 0.0225$ in (a)–(d), respectively. While for the bottom row, fix $\sigma_1 = 0.004$ and let λ increase from left to right: $\lambda = 0.01, 0.2, 0.5, 5.0$ in (e)–(h), respectively.

3. Pattern formation induced by correlated noise in network

3.1. Case I. $R = 0.0$

In this case, the lattice is only affected by the local Gaussian colored noise. The results in the top row of Fig. 1 show some snapshots of the spatial grid at given times n for various σ when λ is taken as 0.05. Let $\sigma_1 = \sqrt{\sigma}$. For $\sigma_1 = 0.0039$, as shown in Fig. 1(a), local noise is not strong enough to excite the system to evoke any ordered spatial structure. But if we increase σ_1 a little to 0.004, the numerical result shown in Fig. 1(b) indicates that noise-induced circular waves emerge in this spatially extended system. If we continue to increase σ_1 , the circular wave will break down (see Fig. 1(c)), where $\sigma_1 = 0.007$. When σ_1 increases to a larger value $\sigma_1 = 0.0225$, the beautiful ordered spatial pattern will give way to strong noisy perturbations, yielding a disordered looking spatial portrait (see Fig. 1(d)).

Next, we fix $\sigma_1 = 0.004$ and let λ change to study the influence of the correlation time of local colored noise on the lattice. It can be seen from the bottom row of Fig. 1 that there exist some intermediate values of λ for which the lattice shows ordered circular wave patterns (see Fig. 1(f) and (g)). While for smaller and larger λ , the network does not form ordered spatial patterns (see Fig. 1(e) and (h)).

3.2. Case II. $0 < R < 1.0$

Because all the local units we consider here are identical and with the same initial conditions, they will behave as a single unit with only common Gaussian white noise if $R = 1.0$. Thus, we just consider $0.0 < R < 1.0$ for correlated noise. Here we set $\lambda = 0.05$.

At first, we take $\sigma_1 = 0.0039$, at which the local colored noise can not fire the lattice alone as shown in Fig. 2(a). For this noise intensity, there exists intermediate R at which the lattice shows ordered circular wave patterns (see Fig. 2(b)). While for $\sigma_1 = 0.0041$, the lattice can fire under the only effect of the local Gaussian colored noise as shown in Fig. 2(d). For this noise intensity, the ordered circular waves are broken down with increasing R . For $\sigma_1 = 0.001$, the correlated noise is not able to fire the lattice, even though we increase the value of R further (diagram not shown here).

From the above simulation results, we can conclude that: (i) colored noise can induce ordered patterns at intermediate noise intensity σ or correlation time λ^{-1} ; (ii) for intermediate noise intensity, the correlated noise can induce ordered patterns only at intermediate noise correlation R ; (iii) however, for larger noise intensity the ordered patterns are blurred by the correlated noise. The obtained ordered circular wave patterns given in Fig. 1(b), (g) and Fig. 2(b), (e) indicate the occurrence of STCR. In the following section, we will analyze the spatial pattern formation quantitatively.

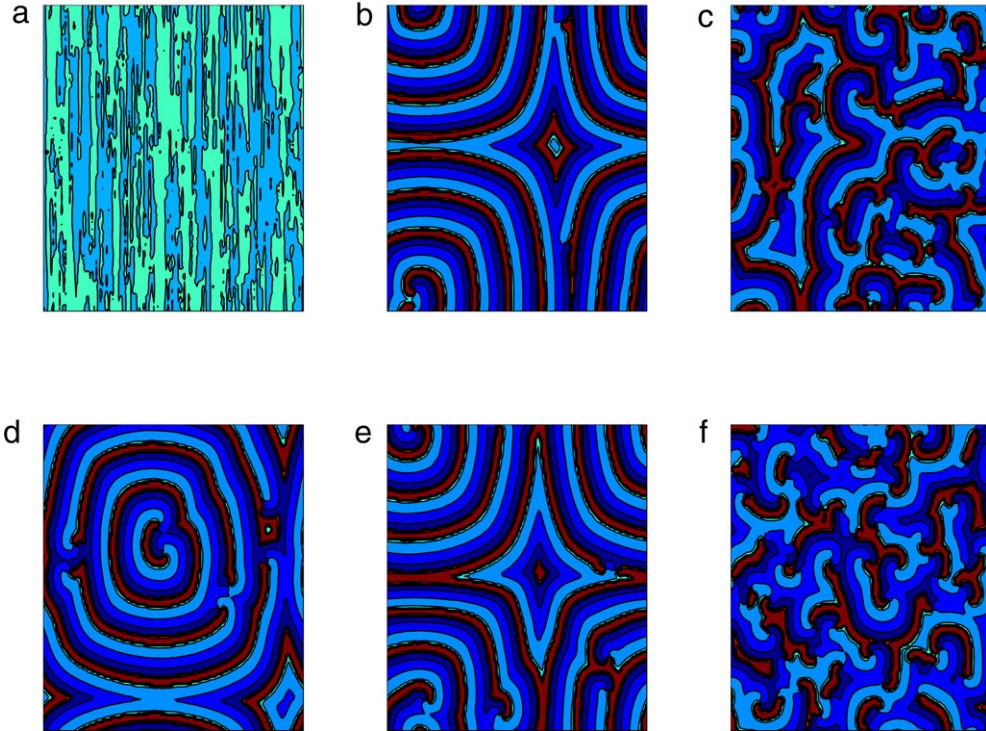


Fig. 2. Snapshots of $u(i, j)$ at grid points of the lattice with correlated noise at $D = 0.0025$ and $\lambda = 0.05$. All snapshots of the spatial grid are taken at given times n . The noise correlation R increases from left to right. For the top row, fix $\sigma_1 = 0.0039$ and let $R = 0.0, 0.01, 0.04$ in (a)–(c), respectively. While for the bottom row, fix $\sigma_1 = 0.0041$ and let $R = 0.0, 0.01, 0.1$ in (d)–(f), respectively.

4. Spatiotemporal coherence resonance

In order to quantify the formation of those spatial structures, we use a linear spatial cross-correlation measure S [27,29]. The cross-correlation S is computed for the variable $u(i, j)$ as the space and time averaged nearest-neighbor distance of all elements, normalized by the total spatial amplitude variance, and is given by

$$S = \left\langle \frac{\text{Cov}(n)}{\text{Var}(n)} \right\rangle_T \quad (6)$$

where the bracket $\langle \rangle_T$ denotes averaging over the total iteration time T . $\text{Var}(n)$ is the spatial variance at the iterated time n given as

$$\text{Var}(n) = \frac{1}{N^2} \sum_{ij} [u(i, j) - \bar{u}]^2 \quad (7)$$

where $\bar{u} = N^{-2} \sum_{ij} u(i, j)$; $\text{Cov}(n)$ is the purely spatial auto-covariance of nearest neighbors, and is defined as

$$\text{Cov}(n) = \frac{1}{N^2} \sum_{ij} \frac{1}{|\mathcal{N}_{ij}|} \sum_{b \in \mathcal{N}_{ij}} [u(i, j) - \bar{u}](b - \bar{u}) \quad (8)$$

with b consisting of all $|\mathcal{N}_{ij}| = 4$ elements of a von Neumann neighborhood \mathcal{N}_{ij} at each lattice site $u(i, j)$.

Obviously, the quantity S is efficient in analyzing nearest-neighbor relationships in space and time. The larger the value of S is, the more coherent the pattern becomes.

4.1. Case I. $R = 0.0$

Corresponding to Section 3.1, we consider the effects of the local Gaussian colored noise on the cross-correlation S of the lattice. For fixed λ , the variation of S with respect to the noise intensity σ (where $\sigma = \sigma_1^2$) is shown in Fig. 3(a). For each λ , S can attain its maximum value at some intermediate noise intensity σ_1 . As for smaller or larger noise intensity, S takes smaller values. This is the typical resonance-like behavior with the variation of the noise intensity for each λ . It is also seen that S remains almost constant for smaller σ_1 .

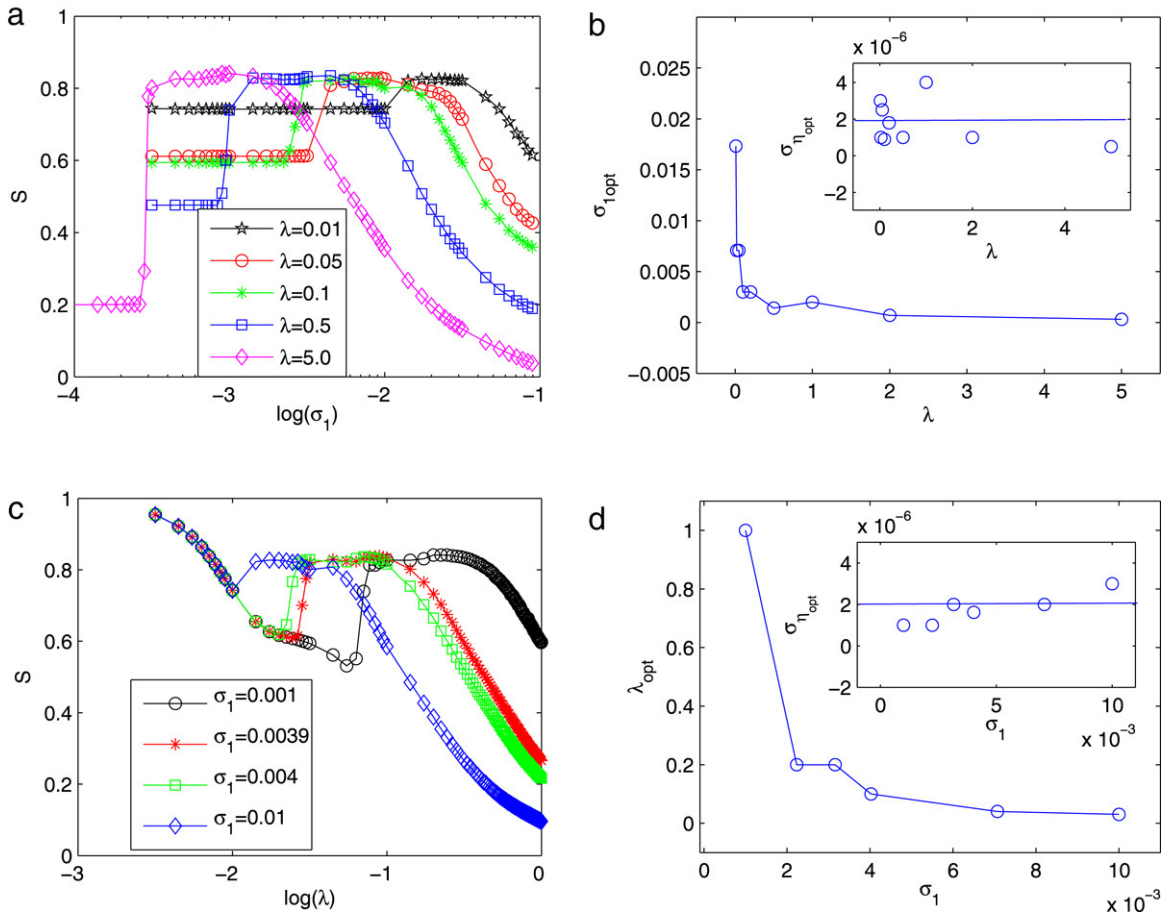


Fig. 3. Spatial cross-correlation S with respect to σ_1 (or λ) for various λ (or σ_1) are shown in (a) (or (c)). It is shown that S exhibits typical resonant-like behaviors with the variation of σ_1 (or λ). Dependence of the optimal value of σ_1 (or λ), corresponding to the maximum of S , on different values of λ (or σ_1) are shown in (b) (or (d)). The optimal value of the noise strength σ_η , which remains almost constant for different values of λ (or σ_1), are shown in the insets of (b) (or (d)).

For fixed σ_1 , this resonance-like behavior can also be observed with the variation of λ (Fig. 3(c)). It is seen that S takes a larger value at smaller λ in Fig. 3(c). Larger S at smaller λ is caused by the interplay of the correlation time λ^{-1} and the characteristic time of the slow variable ν of the local system. For smaller λ , the correlation time λ^{-1} of the colored noise is near the characteristic time of the slow variable ν of the Rulkov system. Thus, the units in the lattice are more correlated with each other; in turn the value of S is larger even though they are not firing. As for larger λ , the correlation time is far from the characteristic time of the slow variable ν , and then the value of S is smaller. In such way the variation of S shows resonance-like behavior.

Furthermore, we can see that $\sigma_{1,opt}$ and λ_{opt} needed for inducing coherent patterns increase with the decreasing of λ and σ_1 , respectively (see Fig. 3(b) and (d)). Now we consider the noise strength of the colored noise given by $\sigma_\eta = \sigma_1^2 \lambda$. If we calculate $\sigma_{\eta_1} = \sigma_{1,opt}^2 \lambda$ and $\sigma_{\eta_2} = \sigma_1^2 \lambda_{opt}$, we find that they fluctuate around 2×10^{-6} , keeping nearly constant (see the inserted figures in Fig. 3(b) and (d), respectively). Therefore, it is concluded that the optimal value $\sigma_{\eta,opt}$ of the noise strength for maximum coherence of the lattice can be about 2×10^{-6} , independent of the correlation time of local colored noise.

4.2. Case II. $0.0 < R < 1.0$

Corresponding to Section 3.2, we consider the effects of the correlated noise and calculate S with respect to the noise correlation R for different values of σ_1 (where $\sigma = \sigma_1^2$) (see Fig. 4). We still take $\lambda = 0.05$ here. For $\sigma_1 = 0.001$, S does not change much for $0 < R < 1$. If σ_1 increases to 0.0039, S takes a maximum value at some intermediate noise correlation R . While for larger $\sigma_1 = 0.0041$, S monotone decreases with R increasing. And S decreases more rapidly with the further increasing of σ_1 .

In order to illustrate these phenomena, we turn back to Eq. (5). For $\lambda = 0.05$, we have $2\sigma > \sigma\lambda$. In this case, increasing R will increase the noise strength (the total power) of the correlated noise $\eta(i, j)$ for fixed noise intensity σ . For smaller noise

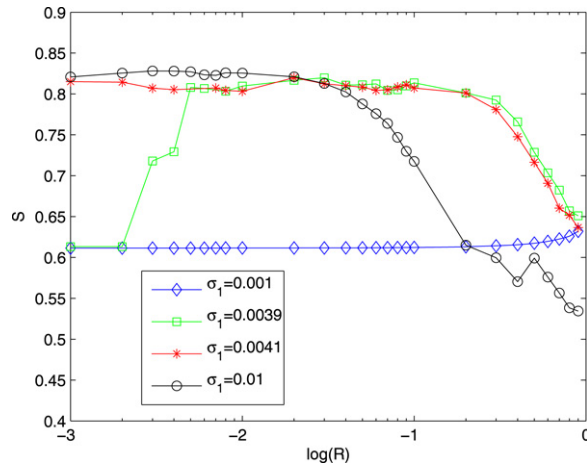


Fig. 4. Spatial cross-correlation S shows resonant behavior with respect to R for intermediate $\sigma_1 = 0.0039$. For larger $\sigma_1 = 0.0041$ and $\sigma = 0.01$, S decreases with increasing R . For smaller $\sigma_1 = 0.001$, S does not change so much.

intensities, the noise strength σ_η is not strong enough to let the lattice fire. Hence, S remains almost invariant for $0 < R < 1$ and the coherence of the lattice has little change. For intermediate noise intensity, the noise strength σ_η will increase with R increasing and pass through the optimal value $\sigma_{\eta\text{opt}}$. Thus, there exists an optimal $R \approx 0.03$, corresponding to the optimal noise intensity $\sigma_{\eta\text{opt}} = 2R\sigma + (1-R)\sigma\lambda \approx 2 \times 10^{-6}$, where the coherence of the lattice reaches its maximum. For a larger noise intensity, increasing R will make the noise strength σ_η far from the optimal value $\sigma_{\eta\text{opt}}$. Thus, the coherence of the lattice decreases with increasing R .

From the above simulation, we can see that the optimal noise strength does not depend noticeably on the correlation time of the colored noise. This is due to the characteristic time of the fast variable u being much smaller than the correlation time of the colored noise. Thus, the correlation time of the colored noise has little effect on the optimal noise strength of the correlated noise. Next we discuss the existence of the optimal noise strength, that is, the mechanism of STCR observed before. For smaller noise strength, the noise is not strong enough to induce any patterns and the coherence is smaller. For intermediate noise intensity, some neurons fire at first and act as initiators to make waves travelling through the whole lattice, so coherence is strengthened. For larger noise intensity, neurons in the lattice compete with each other to act as initiators, leading to decoherence. This is the STCR phenomenon occurring in the lattice.

5. Summary and conclusion

The Rulkov map is employed to model the behavior of neurons with self-sustained subthreshold oscillations. This paradigmatic model also replicates the dynamics of spiking-bursting activity of real biological neurons. The use of low-dimensional model maps can be useful for understanding dynamical mechanisms in real neuron ensembles because it shows typical restructuring of collective behavior and is simple enough to study the reasons behind such restructuring.

In this paper, the effects of additive noise on pattern formation and STCR are investigated in a square lattice network locally modelled by the Rulkov map. It is found that local colored noise is able to induce STCR at intermediate noise intensity and correlation times of colored noise. By adding common Gaussian noise, it is also found that the correlated noise can also induce pattern formation and spatiotemporal coherence resonance for intermediate noise intensity, but this is not the case for smaller and larger noise intensities. Meanwhile, for larger noise intensities, the noise correlation decreases the coherence of noise-induced patterns. This destructive effect of noise correlation on coherence has also been obtained but for rather different systems in Refs. [18,19]. The effects of colored noise on structure formation in excitable media and oscillatory neural models were discussed in Refs. [29,30], respectively. Both of them showed that the optimal noise strength for STSR decreases with increasing noise correlation time. But we find here that the optimal noise strength remains almost constant, which is not affected by the noise correlation time. The difference can be explained as follows. In [29,30], the colored noise acts as white noise because the correlation time of the colored noise considered is smaller than the characteristic time of the local system. Thus the optimal noise intensity remains constant and then the optimal noise strength decreases with increasing the correlation time of the colored noise. In this paper, the correlation time of colored noise is much larger than the characteristic time of the fast variable of the local Rulkov system, thus the optimal noise strength remains constant by changing the correlation time.

Except for the noise correlation, spatial heterogeneity is also a significant ingredient in a spatially extended system. Recently, there are several reports about the effect of diversity on spatially extended systems. It was found that parameter variability is able to induce pattern formation [31] and resonance-like behavior [32] in an excitable square lattice. For spatially extended chaotic systems, it was also found that parameter variability has the ability to induce coherence

resonance [33]. Except for these reports, the parameter diversity has been found as a source of self-sustained information flow in excitable arrays [34], and also that it can act as a mechanism for stochastic resonance in generic soft matter systems [35]. Furthermore, effects of noise on the variability of noise-induced phenomena have also been discussed in Refs. [32,36]. In forthcoming studies, further attention should be paid to the effect of parameter diversity on the noise-induced spatiotemporal coherence resonance in the studied network.

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