



ONSET OF PHASE SYNCHRONIZATION IN NEURONS WITH CHEMICAL SYNAPSE

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We study the onset of synchronous states in realistic chaotic neurons coupled by mutually inhibitory chemical synapses. For the realistic parameters, namely the synaptic strength and the intrinsic current, this synapse introduces noncoherences in the neuronal dynamics, yet allowing for chaotic phase synchronization in a large range of parameters. As we increase the synaptic strength, the neurons reach a periodic state, and no chaotic complete synchronization is found.

Keywords: Chaotic phase synchronization; neuron dynamics; dynamical coupling.

1. Introduction

Many neural networks relay on a balanced configuration of electrical and chemical synapses for a normal functioning. The electrical synapse is usually associated to processes that require rapid responses, since the synaptic delay can be neglected. The chemical one is mediated by chemical transmitters and it is usually associated to processes that do not require rapid responses, since there is an intrinsic synaptic delay.

Neural networks with electrical synapse, as well as the analogous linearly coupled oscillators, have recently attracted much attention mainly because it provides a simple and clear scenario for the onset of synchronization. The basic idea behind this is that for interacting neurons and oscillators with electrical coupling, increasing the coupling strength leads to synchronous behavior. This relation is important since the more synchronous the oscillators are, the more information between themselves can be exchanged [Baptista & Kurths, 2005].

On the other hand, this relation to neurons coupled via chemical synapse is still unclear. In such a synapse, increasing its strength might change the neuronal dynamics, since the synapse itself is a dynamical system. This relation becomes even more complex if the chemical synapse is of the inhibitory type. In that case, while one neuron spikes the synapse forces the other neuron not to spike.

In electrical coupling, several types of synchronization were found in coupled chaotic oscillators — Complete synchronization [Fulisaka, 1983], generalized synchronization [Rulkov *et al.*, 1995]. There is a type of synchronization which appears for very small coupling strength — the phase synchronization (PS) where the coupled chaotic oscillators have their absolute phase difference bounded but their amplitudes may be uncorrelated [Rosenblum *et al.*, 1996]. This was numerically seen in a variety of coupled oscillators [Boccaletti *et al.*, 2002]. These many types of synchronization were also found in neurons [Elson *et al.*, 1998; Wang *et al.*, 2005] with electrical

synapses. In particular, PS was found in two electrically coupled neurons [Shuai & Durand, 1999] and in small neural networks [Wang & Lu, 2005].

The purpose of this work is to analyze the inhibitory chemical synapses in coupled chaotic neurons, and its role for synchronization. We show that there is no complete chaotic synchronization, since as the coupling increases the neurons reach periodic states. This offers a great contrast to synapses of the electrical type in which complete chaotic synchronization is commonly found. We also show that this inhibitory synapse is responsible for introducing phase synchronous behavior, for a wide range of parameters. This result is biologically meaningful since the onset of phase synchronization provides a good environment for communication with chaotic systems since in phase synchronous states one can send information with low probability of errors [Baptista & Kurths, 2005].

This paper is organized as follows. In Sec. 2, we introduce the realistic Hindmarsh–Rose neuron model and in Sec. 3, we introduce the model for the inhibitory chemical synapses. In Sec. 4, we show the likely occurrence of phase synchronization in two neurons with chemical synapse and in Sec. 5, we present the conclusions.

2. Neuron Model

The neurons are described by the 4D Hindmarsh–Rose model which consists of four coupled differential equations [Pinto *et al.*, 2000]

$$\begin{aligned}\dot{x} &= ay + bx^2 - cx^3 - dz + I \\ \dot{y} &= e - y - fx^2 - gw \\ \dot{z} &= \mu(-z + S(x + H)) \\ \dot{w} &= \nu(-kw + r(y + l))\end{aligned}\quad (1)$$

This model has been shown to be realistic, since it reproduces the membrane potential of biological neurons [Johnson *et al.*, 1992], and it is able to replace a biological neuron in a damaged biological network, restoring its natural functional activity [Mulle *et al.*, 1986], it also reproduces a series of collective behaviors observed in a living neural network [Pinto *et al.*, 2000]. We integrate the Hindmarsh–Rose model using a Runge–Kutta of order 6 with adaptative step, and set the parameters to obtain a spiking/bursting dynamics. The parameters of the model are: $a = 1.0$, $b = 3.0$, $c = 1.0$, $d = 0.99$, $e = 1.01$, $f = 5.0128$, $g = 0.0278$, $H = 1.605$, $k = 0.9573$, $l = 1.619$, $\mu = 0.0021$, $\nu = 0.0009$, $r = 3.000$, $S = 3.966$.

3. Synapse Model

Each synaptic connection between the neurons is modeled by a nonlinear differential equation that mimics the release of neurotransmitters at the synaptic cleft and its absorption in the post synaptic cell [Sharp *et al.*, 1996]. The current I_{syn} injected in the postsynaptic cell is determined by the dimensionless, scaled synaptic activation $S(t)$.

$$I_{\text{syn}}(t) = g_{\text{syn}}S(t)[x(t) - V_{\text{rev}}] \quad (2)$$

$$\tau \frac{dS}{dt} = \frac{S_{\infty}(V_{\text{in}}(t)) - S(t)}{S_0 - S_{\infty}(V_{\text{in}}(t))}, \quad (3)$$

where V_{rev} is the synaptic potential, V_{in} is the presynaptic voltage, $x(t)$ represents the membrane potential of the postsynaptic neuron, and τ is the timescale governing receptor binding. S_{∞} is given by:

$$S_{\infty}(V) = \begin{cases} \tanh \frac{V - V_{\text{th}}}{V_{\text{slope}}}, & \text{if } V > V_{\text{th}} \\ 0 & \text{if } V \leq V_{\text{th}} \end{cases} \quad (4)$$

We set the parameters of the synapse equations in order to present an inhibitory effect. That is done by using the following parameters: $V_{\text{th}} = -0.80$, $V_{\text{slope}} = 1.00$, $V_{\text{rev}} = -1.58$, and $S_0 \geq 1$.

4. Phase Synchronization

The condition for PS can be written as

$$|\phi_1 - q\phi_2| \leq c, \quad (5)$$

where $\phi_{1,2}$ are the phases calculated from a projection of the attractor onto appropriate subspaces. The neurons present a noncoherent dynamics due to the two time scales, i.e. bursting/spiking behavior. By noncoherent dynamics we mean that there is no clear center of rotation in which the trajectory spirals around and also it is not possible to define a Poincaré section for which the trajectory crosses only once each time the neuron has a hyperpolarization.

The chemical synapse introduces even more noncoherence. This happens because when one neuron is in a spiking behavior it inhibits the other neuron, which might hyperpolarize, but the neuron that has been inhibited still tries to spike. This competition generates more noncoherence in the phase space. As a consequence, it is rather unclear how one can calculate the phases for such dynamics. However, it is possible to overcome this problem by using the conditional Poincaré map [Baptista *et al.*, 2005], which is a map of the attractor, construct by

observing it for specific times at which events occur in one neuron. Using such technique, we can detect PS without actually measuring the phase.

4.1. PS-sets

The conditional Poincaré map is a map of the flow. In particular, it consists in observing the trajectory of the neuron \mathcal{N}_j at special times τ_j^i , with the index $j = 1, 2$ indicating the two neurons. We define these times of events τ_j^i , by the following rule:

- τ_1^i represents the time at which the membrane potential in the neuron \mathcal{N}_2 reaches a threshold for i th times.
- τ_2^i represents the time at which the membrane potential in the neuron \mathcal{N}_1 reaches a threshold for i th times.

Then, we record the trajectory position of the neuron \mathcal{N}_j at these times τ_j^i . As a result, we have a discrete set of points called \mathcal{D}_j . If \mathcal{D}_j does not spread over the attractor of \mathcal{N}_j , but is rather localized, we say that the set \mathcal{D}_j is a PS-set. It can be shown that PS-set implies PS [Baptista *et al.*, 2005]. This is so, because the difference between the time at which the i th event happens in both oscillators is small, which means that the time difference $|\tau_1^i - \tau_2^i| < \delta$, with δ being a small constant. As a consequence, the points in the conditional Poincaré map are confined.

In Fig. 1 we show two types of \mathcal{D}_j set. In (A), the \mathcal{D}_j is a PS-set. One can see that this

set is localized, and so it does not spread over the attractor, then we have phase synchronization. The parameters are $I = 3.12$ and $g_{\text{syn}} = 0.78$. In (B), is a situation where there is no phase synchronization, for $I = 3.12$ and $g_{\text{syn}} = 0.76$. The set \mathcal{D}_j spreads over the attractor. To have a global view of the possible behaviors in this system, in Fig. 2 we show the parameter space in the coordinates $I \times g_{\text{syn}}$. In this parameter space we depict in color the parameters for which we have phase synchronization, and in white, parameters for which either chaos with no synchronous behavior or periodic states is found. The color bar on the right of this figure indicates the relative area in percentage of the \mathcal{D}_j set occupation in the attractor projection. To assure that we have chaotic phase synchronization we also compute the standard deviation of the event times. We introduce the quantity $T_j^i = \tau_j^i - \tau_j^{i-1}$, so its relative standard deviation is given by $\varsigma = \{\langle (T_j^i)^2 \rangle - \langle T_j^i \rangle^2\} / \langle T_j^i \rangle$, where $\langle \cdot \rangle$ represents the average. In Fig. 3, we show ς as a function of I and g_{syn} . We have two kinds of transitions for phase synchronization. In the first, the neurons are chaotic and present a nonsynchronous behavior, then by increasing the synaptic coupling there is a transition to phase synchronization, fixing the current $I = 3.12$. The first transition occurs around $g_{\text{syn}} \approx 0.77$. In the second transition, the neurons are in a periodic behavior and when we increase the coupling strength they attain chaos but phase synchronized.

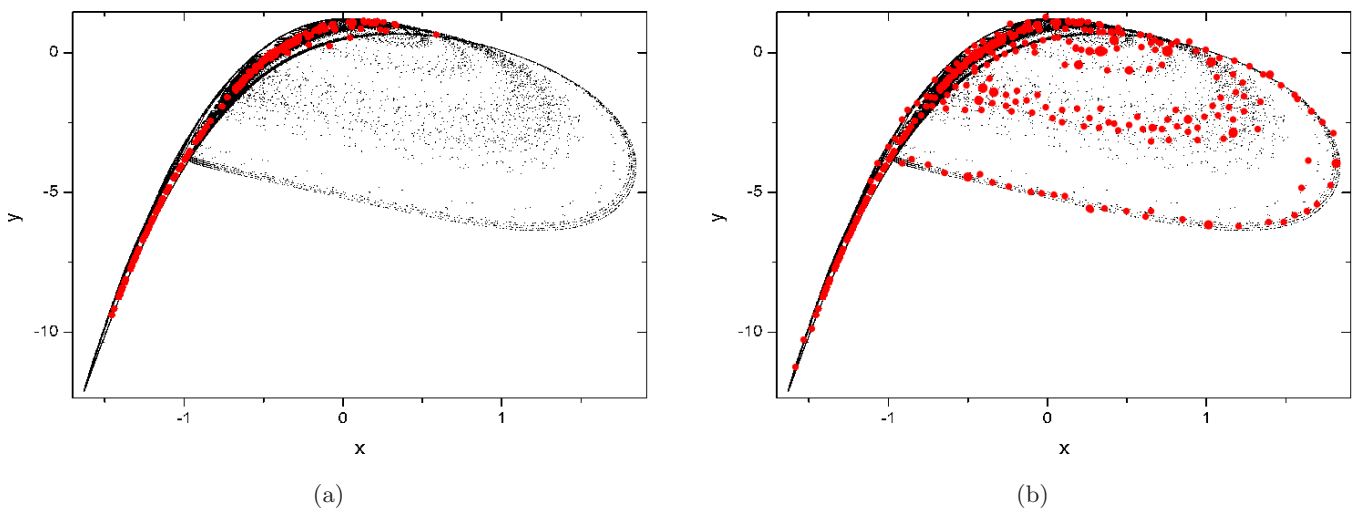


Fig. 1. The attractor is depicted in black and the set \mathcal{D}_j in red. In (a), the set \mathcal{D}_j is localized with respect to the attractor, and the map \mathcal{D}_j does not fulfill the whole attractor projection, occupying just partially this projection. This implies the presence of phase synchronization between the two neurons. The parameters are $I = 3.12$ and $g_{\text{syn}} = 0.78$. In (b), the set \mathcal{D}_j spreads over the attractor and fulfills it showing that there is no PS, the parameters are $I = 3.12$ and $g_{\text{syn}} = 0.76$.

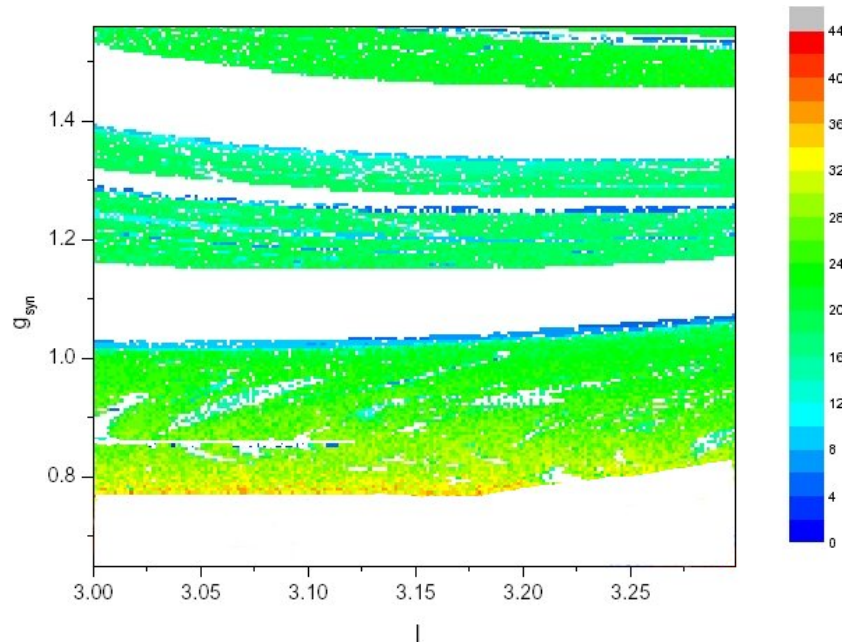


Fig. 2. The parameter space $I \times g_{\text{sync}}$ is depicted. The colored regions represent parameters in which the neurons present PS-sets, which imply phase synchronization. White regions represent either chaotic or periodic behavior.

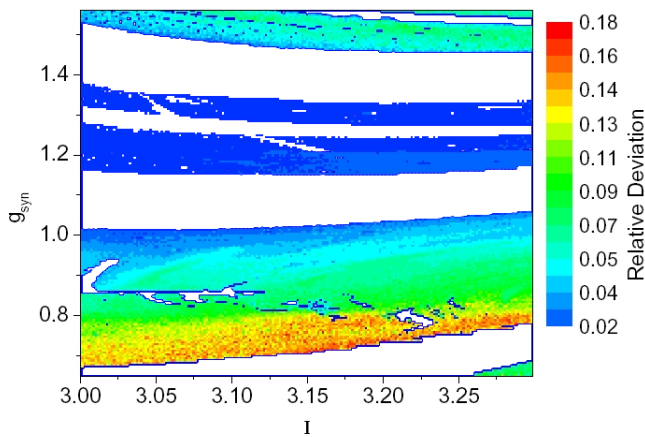


Fig. 3. The parameter space $I \times g_{\text{sync}}$ is depicted. The colored regions represent the relative standard deviation of the burst time for one neuron. A positive relative standard deviation means that the neuron is still chaotic. The greater is the standard deviation, the more chaotic is the neuron. White regions represent periodic behavior.

5. Conclusions

We have shown that phase synchronization is a common behavior in neurons with inhibitory chemical synapses for the Hindmarsh–Rose model. In addition, it is shown that there is no complete chaotic synchronization, since as the synapse strength increases the neurons reach periodic states. This places the inhibitory chemical connection as a good

candidate to explain information transmission processes regulated by means of phase synchronization.

The neurons present naturally noncoherent dynamics due to the two time scales provided by the bursting and spiking dynamics. As they are coupled by a chemical inhibitory coupling, they undergo an even more noncoherent state. However, we could still detect the presence of phase synchronization using the conditional Poincaré map.

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