Predicting thermal displacements in modular tool systems

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In the last decade, there has been an increasing interest in compensating thermally induced errors to improve the manufacturing accuracy of modular tool systems. These modular tool systems are interfaces between spindle and workpiece and consist of several complicatedly formed parts. Their thermal behavior is dominated by nonlinearities, delay and hysteresis effects even in tools with simpler geometry and it is difficult to describe it theoretically. Due to the dominant nonlinear nature of this behavior the so far used linear regression between the temperatures and the displacements is insufficient. Therefore, in this study we test the hypothesis whether we can reliably predict such thermal displacements via nonlinear temperature-displacement regression functions. These functions are estimated first from learning measurements using the alternating conditional expectation (ACE) algorithm and then tested on independent data sets. First, we analyze data that were generated by a finite element spindle model. We find that our approach is a powerful tool to describe the relation between temperatures and displacements for simulated data. Next, we analyze the temperature-displacement relationship in a silent real experimental setup, where the tool system is thermally forced. Again, the ACE algorithm is powerful to estimate the deformation with high precision. The corresponding errors obtained by using the nonlinear regression approach are 10-fold lower in comparison to multiple linear regression analysis. Finally, we investigate the thermal behavior of a modular tool system in a working milling machine and again get promising results. The thermally induced errors can be estimated with 1–2 μm accuracy using this nonlinear regression analysis. Therefore, this approach seems to be very useful for the development of new modular tool systems. [DOI: 10.1063/1.1622351]

Manufacturing processes are highly complex systems—predicting, controlling and optimizing of such processes is rather difficult. A major challenge is an optimum design of the manufacturing processes to consistently achieve quality targets. Here we investigate especially thermal influences on machine tools which are essential to get stabilized manufacturing processes with tolerances in the micrometer range. The modular tool systems, considered in this study, are interfaces between spindle and workpiece and consist of several complicatedly formed parts. Their thermal behavior is dominated by nonlinearities, delay and hysteresis effects even in tools with simpler geometry and it is difficult to describe it theoretically. Due to the dominant nonlinear nature of this behavior the so far used linear regression between the temperatures and the displacements is insufficient. Therefore, we introduce a concept of maximal correlation which is a very powerful criterion to measure the dependence of two especially nonlinear related variables. In this study, we show that we can significantly improve the prediction of thermal displacements via nonlinear temperature-displacement regression functions, which are estimated using the alternating conditional expectation algorithm, a recently developed approach of nonlinear time series analysis. The practical applicability of this method was proven on a working milling machine—the thermally induced errors could be estimated with 1–2 μm accuracy. Therefore, this approach seems to be very useful not only for the development of new modular tool systems but also for other interdisciplinary applications.

I. INTRODUCTION

In the recent decade, much attention has been devoted to the investigation of the thermal influence on complete machine tools to get thermally stabilized processes with tolerances in the micrometer range.[1–12] Modular tool systems as interfaces between spindle and workpiece, however, affect to a great extent the accuracy of machining. Therefore, the thermal behavior of modular tool systems has to be thoroughly investigated, especially using nonlinear-dynamical approaches, which turned out to be very promising in engineering.[13,14]

Economic and ecological reasons lead to so-called "dry

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processing'' without the use of any lubricants and coolants. However, the stronger thermal influence in dry processing due to friction causes displacements up to 100 μm. Over the past few decades, the compensation for defects due to a thermal impact has attracted steadily increasing interest. These investigations comprise a wide range of methods ranging from computer simulations[15] and internal monitoring in neural networks[16,17] to thermal error modeling.[2,3,7,15,18,19] The relation between temperatures and displacements is very complex even in simple tools.[9] A modular machine tool consists of several complicatedly formed interfaces. Therefore, it is difficult to describe thermal displacements theoretically. The state of the art in the description of thermal displacements is represented today by a linear regression between the temperatures and the displacements.[20] One disadvantage of this approach is that it cannot describe hysteresis behavior, especially for very fast heating or cooling. In that case the temperatures measured on the surface of the modular tool systems are different from the real temperatures inside the system, which leads to time delays in the temperature propagation.

The purpose of this paper is, therefore, to use nonlinear regression methods to model the relation between temperatures and inelastic displacements and finally to predict the thermal displacements quantitatively in a high precision. The paper is organized as follows. In Sec. II we shortly describe the nonlinear regression approach we are using. In Sec. III we present the application of this approach to data that were generated by a finite element spindle model. Section IV contains the application to a silent real experimental setup, where the tool system was thermally forced. In Sec. V we investigated the thermal behavior of a modular tool system in a working milling machine and, finally, in Sec. VI we discuss our results.

II. MAXIMAL CORRELATION AND OPTIMAL TRANSFORMATIONS

We generally assume that there are measurements T_i, $i=1,\ldots,n_i$ of the temperature and S_j, $j=1,\ldots,n_j$ of the axial displacement at n_i, resp., n_j points (e.g., Fig. 1). We use these different measurement points, e.g., at the cutting edge, to describe the thermal process in the whole modular tool system. The main aim is then to reconstruct the displacements S_j on the basis of manufacturing parameters, especially the temperatures T_i. Because of well-known hysteresis effects,[9] it is necessary to consider a response transformation model, which is of the type

$$\theta_j(S_j)=\phi_j(T_1,\ldots,T_{n_i}), \quad j=1,\ldots,n_j. \tag{1}$$

The regression functions ϕ_j are high dimensional surfaces and cannot be displayed for dimensions greater than 2. Moreover, they do not provide a geometrical description of the regression relationship between the temperatures and the displacements. To overcome this problem, we consider the following models:

$$\theta_j(S_j)=\sum_{i=1}^{n_i}\phi_{j,i}(T_i), \quad j=1,\ldots,n_j, \tag{2}$$

and apply the alternating conditional expectations (ACE) algorithm,[21] described below, as a nonparametric approach to estimate the transformations $\phi_{j,i}$ and θ_j in Eq. (2).

The concept of maximal correlation is a very powerful criterion to measure the dependence of two especially nonlinear related variables.[22] The main idea of this approach is to measure the maximized correlation of properly transformed variables.

Given a real variable S_j and an n_i-dimensional vector $T=(T_1,\ldots,T_{n_i})$ in the additive model (2). Then, the maximal correlation is defined by

$$\Psi_j(S_j,T):=|\rho(\theta_j^*(S_j),\phi_j^*(T))|=\max_{\theta,\phi}|\rho(\theta(S_j),\phi(T))|, \tag{3}$$

where ρ denotes the correlation coefficient. The functions θ_j^* and ϕ_j^*, which fulfill the maximal condition (3), are called optimal transformation and represent an estimation of the model (2). To estimate them nonparametrically, we use the ACE algorithm.[21] This iterative procedure is nonparametric because the optimal transformations are estimated by local smoothing of the data using kernel estimators. We use a modified algorithm in which the data are rank-ordered before the optimal transformations are estimated. This makes the result less sensitive to the data distribution. For more details see Appendix A.

The maximal correlation and optimal transformation approach have been recently applied to nonlinear dynamical systems especially to identify a delay in lasers[23] and partial differential equations in fluid dynamics.[24] The ACE algorithm turned out to be a very efficient tool for nonlinear data analysis.[23,25,26]

III. RECONSTRUCTION OF THERMALLY INDUCED DISPLACEMENTS IN A FINITE ELEMENT SPINDLE MODEL

To investigate whether the nonlinear regression approach described above is appropriate also for modular tool systems, which are used in milling and drilling machines, we first analyze simulated data from finite element models (FEM). The tool system is the connecting part between the main spindle and the milling tool. Its main target is the production of clamping forces for an accurate machining. There are different types of clamping systems; in this work we focus on power shrinking and hydraulic chucks tools. These data sets obtained with the FEM model are used to find an optimal number and optimal locations of measurement points and to study the influence of the controlling parameters. In the FEM model, a given regime of rotations leads to the temperatures T_i and the displacements S_j at several measurement points. In the following, we are investigating a main spindle tool design (Fig. 1) with two different simulations. We use the first measurement as a learning set $\{S_{t,j}\}_{t=1,\ldots,n}$, $j=1,\ldots,n_j$, $\{T_{t,i}\}_{t=1,\ldots,n}$, $i=1,\ldots,n_i$ to compute the optimal transformations and the second as a test series $\{S'_{t,j}\}_{t=1,\ldots,n}$, $j=1,\ldots,n_j$, $\{T'_{t,i}\}_{t=1,\ldots,n}$, $i=1,\ldots,n_i$ to check how well the optimal transformations obtained by the reference series describe the temperature-displacement relation. The only difference between both series is given with

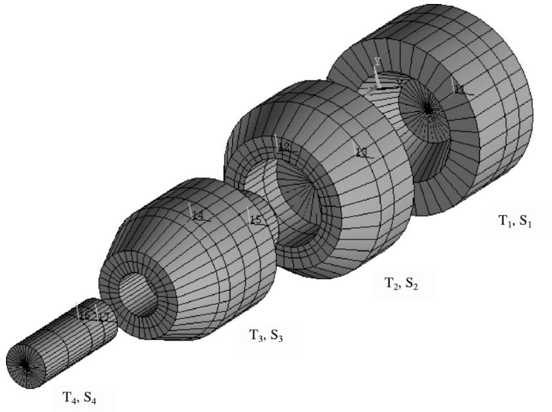

FIG. 1. Design of the main spindle model (output of the commercial FE-modeling program ANSYS).

different manufacturing schemes, especially with different time scales and regimes of rotations. We apply a two step strategy.

(i) Computation of the optimal transformations θ_j^* and ϕ_j^* in the model

$$\theta_j(S_{t,j})=\phi_j(T_{t,i_{i\in I}}) \\ j=1,\dots,n_j=\sum_{i=1,\dots,n_i}\phi_{j,i}(T_{t,i}), \tag{4}$$

from the training series $\{S_{t,j}\}_{t=1,\dots,n}$, $j=1,\dots,n_j$, $\{T_{t,i}\}_{t=1,\dots,n}$, $i=1,\dots,n_i$.

(ii) Reconstruction of the displacements $S'_{t,j}$, $j=1,\dots,n_j$ of the test series using the temperatures $T'_{t,i}$, $i=1,\dots,n_i$ of the test series and the optimal transformations θ_j^* and ϕ_j^* of the training series computed in (i),

$$\hat{S}'_{t,j}=\hat{\theta}_j^{*-1}\left(\sum_{i\in I}\hat{\phi}^*_{j,i}(T'_{t,i})\right),\ j=1,\dots,n_j, \tag{5}$$

and a comparison of the real $S'_{t,j}$ with the estimated $\hat{S}'_{t,j}$ displacements.

The hat over θ and ϕ denotes a nonparametric estimation for these functions. Practically, we use a nearest-neighbor-estimation with $k=2$ nearest neighbors[27] to estimate these functions. We have tested different numbers of nearest neighbors $k=1,..,10$, however, there are only slight differences in the estimates.

For two different regimes of rotations we have a simulated data set with values of temperatures ($T_{t,i}$, $i=2,3,4$) and displacements ($S_{t,j}$, $j=3,4$) belonging to different parts of the tool. Each series consists of $n=199$ points with a sampling time of $\Delta t=100$ s. The reference series, which is used as a training set, is shown in Fig. 2(a).

We present here only the estimations for S_4 at the front tip of the tool (Fig. 1) because the displacements at the cutting edge are the most interesting ones. The results are shown in Fig. 2(b), i.e., the estimations for the optimal transformations θ_4^* and $\phi_{4,2}^*,\dots,\phi_{4,4}^*$. The nonlinearity of these functions is a clear evidence that linear regression is not sufficient to model the relation between temperatures and displacements—otherwise the optimal transformations

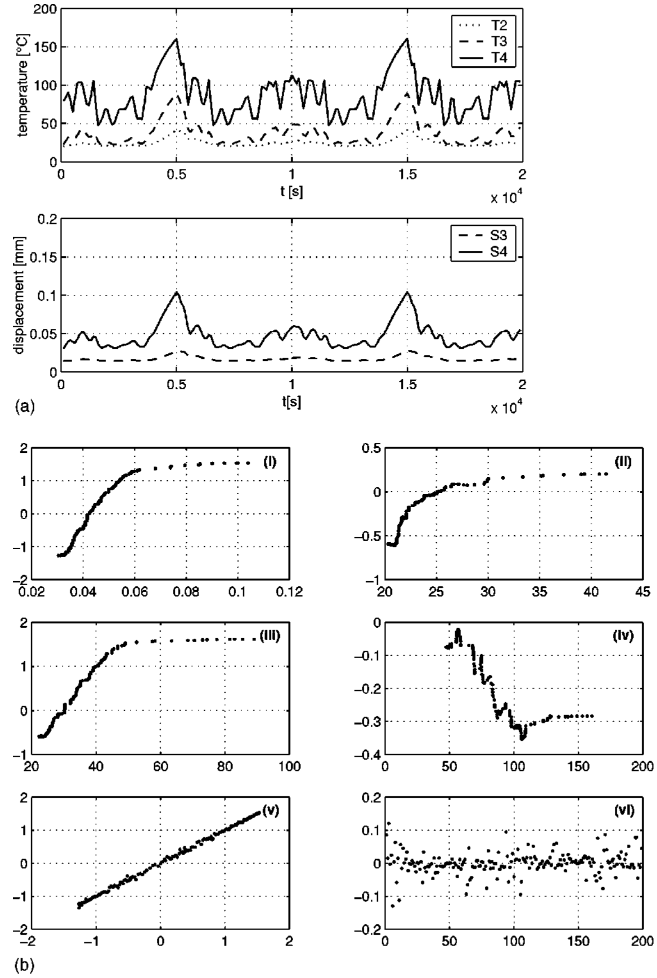

FIG. 2. Simulated data for the model of Fig. 1: (a) Training time series (top: temperature T_2, T_3, T_4, bottom: axial displacements S_3, S_4); (b) optimal transformations (arbitrary units) for S_4 with respect to the model Eq. (4): (i) $\theta_4^*(S_{t,4})$, (ii) $\phi_{4,2}^*(T_{t,2})$, (iii) $\phi_{4,3}^*(T_{t,3})$, (iv) $\phi_{4,4}^*(T_{t,4})$, (v) plotting θ_4^* versus $\Sigma\phi_{4,i}$; (vi) plotting the residuals $\theta_4^*-\Sigma\phi_{4,i}$.

should be close to linear. The graph on the lower left corner 2(b)(v) shows $\theta_4^*(S_{t,4})$ versus $\Sigma_{i=2}^4\phi_{4,i}^*(T_{t,i})$ which should be the identity in the optimal case. In Fig. 2(b)(vi) on the lower right side, we plot the residuals $\theta_4^*(S_{t,4})-\Sigma_{i=2}^4\phi_{4,i}^*(T_{t,i})$. Assuming the existence of functions θ and ϕ in the model, the residuals should vanish and in fact, the high value obtained for the maximal correlation $\Psi=0.9994$ indicates a very good estimation. Furthermore, this value is quite high in comparison with the maximal correlation in other applications of the ACE algorithm.[22,28]

Next, we compute the estimation of displacements of the test data set using the optimal transformations estimated above and the temperatures from the test data set as in Eq. (5). The estimated curves are quite close to the original ones (Fig. 3) except for the cropped peaks. Looking at the original time series and the optimal transformations in Fig. 2, we see immediately that the support in the displacement series of the training set is not as wide as the range of the test series. Therefore, for data from real modular tool systems we have to consider this problem. Moreover, we have to find minimal measurement points with optimal predictability of the displacements to reduce the dimensionality of the task.

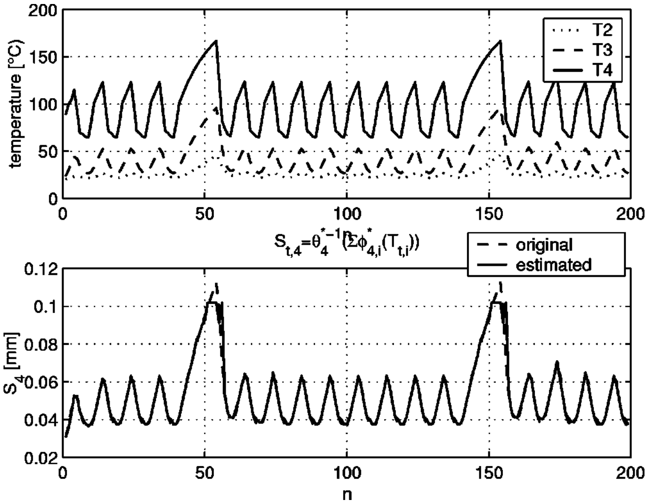

FIG. 3. Estimation of the displacement S_4 for the test time series, top: the original temperatures, bottom: the original and the estimated axial displacements of the main spindle model.

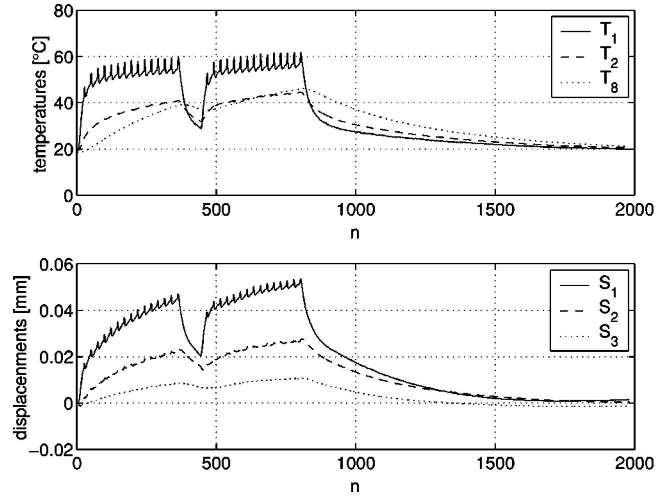

FIG. 5. Measurement for the "resting tool system" (training set), which was thermally forced—T_i, $i=1,2,8$ are the temperatures, S_i, $i=1,..,3$ the measured axial displacements.

IV. THE APPLICATION TO A SILENT REAL EXPERIMENTAL SETUP

In the last section it was possible to demonstrate that the presented nonlinear regression approach is able to predict displacements which were generated using a FEM model. Now, its applicability to measured data from real modular tool systems have to be validated. Therefore, a tool system with a corresponding recording system was designed which is able to measure temperatures and displacements at several tool positions (see Fig. 4). However, as a first step, the tool system was resting, i.e., not active. On a body, a fixed main spindle shaft with the hydraulically actuated tool chucking system HSK 63 is fixed. Electrically driven heating elements enable the intentional introduction of dimensioned heat flows into the tools at the tool chuck via the roller bearing fit of the main spindle shaft. Heating of the cutting edges, which originally results from the cutting procedure, is generated with a shuttered hot air gun in an approximately point-wise manner or at the tool tip with a soldering copper. Then, to avoid external error sources, in a thermal cell temperatures at nine locations and displacements at three locations were simultaneously recorded. In this thermal cell, predefined room and foundations temperatures, including local temperature gradients, can be generated.

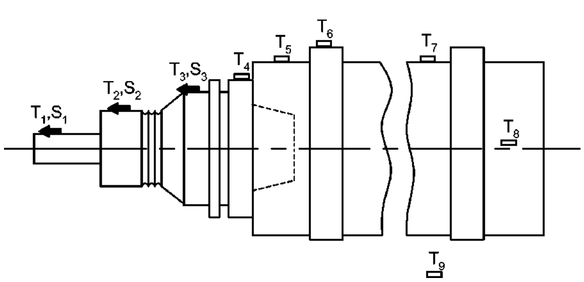

FIG. 4. Scheme of the resting tool system on a fixed shaft with measurement positions S_j, $j=1,..,3$ for the displacements and T_i, $i=1,..,9$ for the temperatures (T_9 is the room temperature).

Let S_j, $j=1,..,3$ denote the measured displacements and T_i, $i=1,..,9$ the temperatures measured. Then, we can assume the following additive model:

$$\theta_j(S_j)=\sum_{i=1}^{9}\phi_{j,i}(T_i), \quad j=1,\ldots,3. \tag{6}$$

As in the previous section we apply a two step strategy: First, the optimal transformations are estimated based on a learning data base. Afterwards, the temperature-displacement relationship is tested on an independent measurement. In this work, only the temperature measuring points T_1, T_2 and T_8 were taken into account for modeling (i.e., $\phi_{j,i}\equiv 0$ for $i\neq 1,2,8$, $j=1,\ldots,3$)—this combination of measuring points has proven to be very efficient in predicting the displacements (the algorithm for the selection of optimal measuring locations is explained in Appendix B). In Figs. 5 and 6, two representative measurements are given. The only difference between the two measurements is a break in the

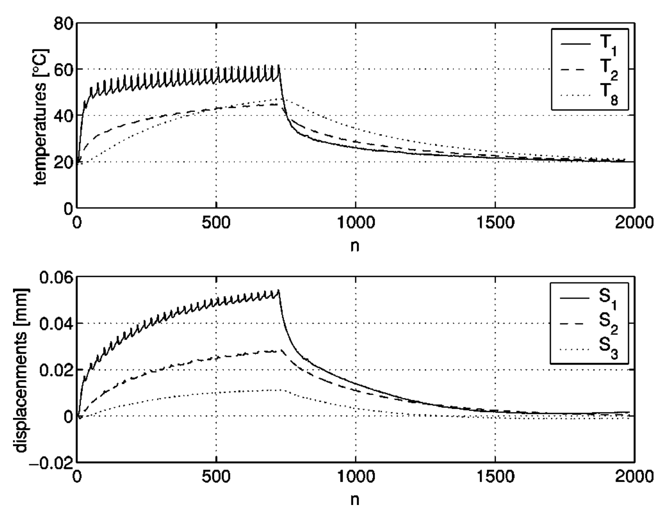

FIG. 6. Measurement for the "resting tool system" (test set)—T_i, $i=1,2,8$ are the temperatures, S_i, $i=1,..,3$ the measured axial displacements.

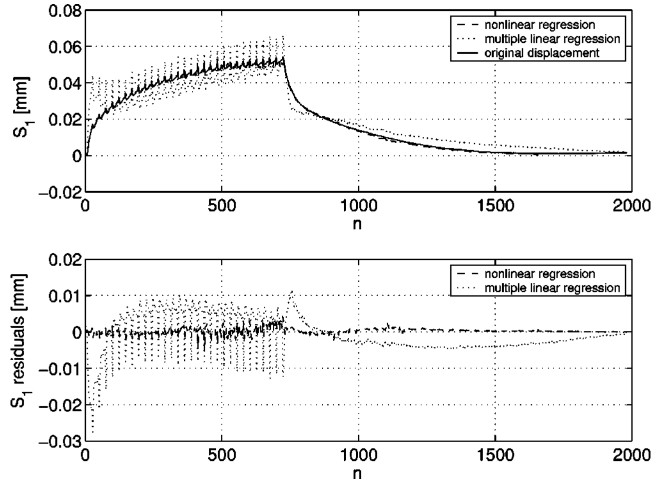

FIG. 7. Estimation of the displacement S_1 of the test set based on the estimated optimal transformations of the training set and the temperatures from the test set (top). On the bottom the residuals to the original displacement S_1 are plotted.

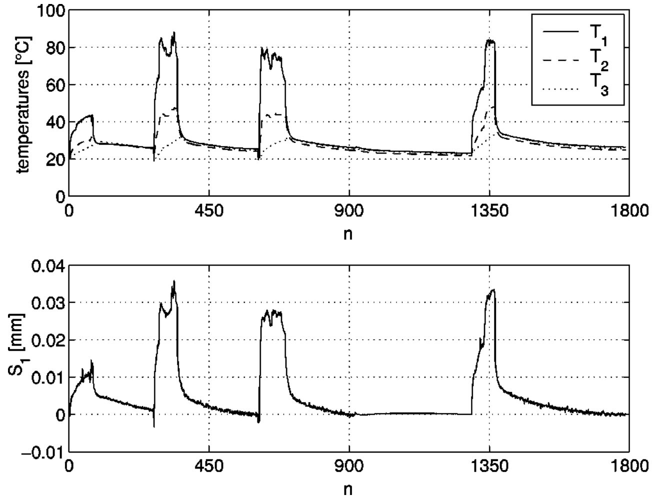

FIG. 8. Measurement for the "working tool system" (training set, four successive measurements)—T_i, $i = 1,2,3$ are the temperatures, S_1 the measured displacement on the tip of the tool system.

neighborhood of the measuring point No. 400 during the measurement for learning (Fig. 5). This break was performed to get a sufficient range of values in the learning series, especially in the fast heating region (30–50 °C).

Figure 7(a) shows the estimated displacements at the tip of the tool S_1 using multiple linear regression (based on the least square method) as well as using our nonlinear regression approach. In Fig. 7(b) the differences to the original measured displacements are plotted. The displacements estimated upon nonlinear regression are very close to the original, the RMS (root mean square) failure is up to 10-fold lower than with multiple linear regression, especially in the active warming region at the beginning of the series. The total RMS error for multiple linear regression amounts to 5.7 μm and to 1.0 μm for nonlinear regression.

V. THE THERMAL BEHAVIOR OF A MODULAR TOOL SYSTEM IN A WORKING MILLING MACHINE

Finally, after successfully applying the nonlinear regression approach to the measured data from a silent experimental setup, we investigate now the thermal behavior of a modular tool system in a working milling machine. In this final section, the thermally caused displacements within the modular tool system were compensated as a demonstrator. On a laptop near the machine, the temperatures measured in the process were entered. Now, using these temperatures, the computer online calculated the displacements based on the temperature-displacement learning set given in Fig. 8. Here, four successive measurements from four different days were taken as the training set to include several manufacturing schemes and finally to have a sufficiently large range of values. It was possible to compare simultaneously the calculated displacements with the measured ones in the cutting procedure. During this demonstration test, the cutting speed values were repeatedly changed in order to generate deviations from the training measurements and, thus, to reinforce the compensating calculations. The cutting speed values varied from $v_c = 120$ m/min, 200 m/min and 370 m/min. For a comparison of the measured displacements and the estimated ones, see Fig. 9. As it can be seen, the nonlinear estimation of the displacement and the original one vary only slightly from each other—the total RMS error is 2.2 μm. Avoiding displacement determination errors as those around measurement point 17 leads to estimations with 1–2 μm accuracy. In contrast, the estimation using multiple linear regression is significantly worse—the RMS error amounts to 5.0 μm. The multiple linear regression tends to a systematic overestimation of the displacements between measurement points 10 and 40, whereas it is underestimated from measurement point 60 on.

VI. CONCLUSIONS

The main purpose of this paper was to test whether we could predict thermal displacements by using a nonlinear

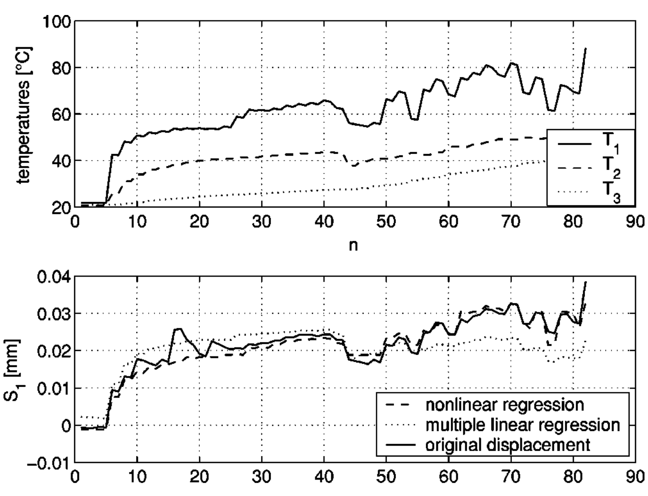

FIG. 9. Thermal error modeling of the test set based on the estimated optimal transformations of the training set (see Fig. 8) and the temperatures from the test set (top). On the bottom the nonlinear and multiple linear estimations of the displacement S_1 as well as the original measured series are plotted.

regression analysis. The data analyzed were generated by a finite element spindle model of modular tool systems, were measured on a silent real experimental setup and finally recorded from a working milling machine. The nonlinear temperature-displacement functions, which are the basis for thermal error modeling, were estimated first from learning measurements using the ACE algorithm and then tested on independent data sets. As it turned out to be very important in this investigation, the support of the training measurements, i.e., the temperature range passed, must be at least as wide as that to be estimated later. Outside the range of values of the training measurements the calculation accuracy is decreasing rapidly. However, in principle, the compensation algorithm described above can be applied to all modular tool systems whose temperature-deformation ratio has been previously determined in a learning trial. Moreover, the algorithm may be immediately transferred to tool systems of similar geometry and consistence. Summarizing the results of this paper, we find that the ACE algorithm is a powerful tool to estimate the transformations θ and ϕ in the analyzed models and thus enables a reliable prediction of thermal displacements. Therefore, this approach seems to be very promising for the development of new modular tool systems.

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APPENDIX A: THE ACE ALGORITHM

In this appendix we provide a short description of the ACE algorithm of Breiman and Friedman,[21] the computer programs used can be obtained from the authors (http://tocsy.agnld.uni-potsdam.de/). In the following the same notations as introduced in Sec. II are used.

Generally, the estimation of functions that are optimal for correlation is equivalent to the estimation of functions that are optimal for regression. Therefore, another writing of the problem,

$$\Psi(S,T_1,\ldots,T_k)=\max_{\theta,\phi_i}\left|\rho\left(\theta(S),\sum_{i=1}^{k}\phi_i(T_i)\right)\right|, \tag{A1}$$

is the regression problem

$$E\left[\left(\theta(S)-\sum_{i=1}^{k}\phi_i(T_i)\right)^2\right]\overset{!}{=}\min. \tag{A2}$$

Here, the functions θ and ϕ_j $(j=1,\ldots,k)$ are varied in the space of Borel measurable functions, and the constraints onto these functions are that they have vanishing expectation and finite variances to exclude trivial solutions.

For the one-dimensional case $(k=1)$, the ACE algorithm works as follows: Denoting the conditional expectation of $\phi_1(T_1)$ with respect to S by $E[\phi_1(T_1)|S]$, then the function $\overline{\phi_0}(S)=E[\phi_1(T_1)|S]$ minimizes (A2) with respect to $\theta(S)$ for given $\phi_1(T_1)$. Similarly, $\overline{\phi_1}(T_1)=E[\theta(S)|T_1]/\|E[\theta(S)|T_1]\|$, where the norm is defined by $\|Z\|=\sqrt{\mathrm{var}[Z]}$, minimizes (A2) with respect to $\phi_1(T_1)$ for given $\theta(S)$, keeping $E[\phi_1^2(T_1)]=1$. Now the ACE algorithm consists of the following iterative procedure: Starting with the initial function

$$\phi_1^{(1)}(T_1)=E[S|T_1], \tag{A3}$$

from $i=2$ one calculates recursively

$$\theta^{(i)}(S)=E[\phi_1^{(i-1)}(T_1)|S] \tag{A4}$$

and

$$\phi_1^{(i)}(T_1)=E[\theta^{(i)}(S)|T_1]/\|E[\theta^{(i)}(S)|T_1]\|, \tag{A5}$$

until $E[(\phi_1^{(i)}(T_1)-\theta^{(i)}(S))^2]$ fails to decrease. The limit values are then estimates for the optimal transformations θ and ϕ_1. For the minimization of the right hand side of Eq. (A2) one uses a double-loop algorithm. In the additional inner loop the functions

$$\phi_j^{(i)}(T_j)=E\left[\theta^{(i)}(S)-\sum_{p\neq j}\phi_p^{(i,i-1)}(T_p)\middle|T_j\right]$$

are calculated. In the sum, the superscript "$.^{(i)}$" is used for $p<j$ and "$.^{(i-1)}$" for $p>j$. For $k>1$ the ACE algorithm works similarly.

There are several possibilities to estimate conditional expectations from finite data sets. In our examples local smoothing of the data is used. This smoothing can be achieved with different kernel estimators. We use a simple boxcar window, i.e., the conditional expectation value $E[y|x]$ is estimated at each site i via

$$\hat{E}[y|x_i]=\frac{1}{2N+1}\sum_{j=-N}^{N}y_{i+j},$$

for a fixed half window size N. In all examples of this paper $N=10$ is used to account for a reliable estimate of the mean value.

Furthermore, to allow for a better estimation in the case of inhomogeneous distributions, prior to the application of the ACE algorithm we transform the data to have rank-ordered distributions [i.e., we sort the data set X in ascending order resulting in the vector Y and all further calculation are performed with the corresponding index vector I, where $Y=X(I)$]. This allows for a more precise estimation of expectation values, independently of the form of the data distribution, and simplifies the algorithm considerably. It is allowed since the rank transformation is invertible and the maximal correlation is, by definition, invariant under invertible transformations. Proofs of convergence and consistency of the function estimates are given in Ref. 21.

APPENDIX B: ALGORITHM FOR SELECTING OPTIMAL MEASURING LOCATIONS

In the silent real experimental setup for three different tool mountings (HSK 63 interface; collet chuck, hydraulic expansion chuck and shrink chuck) the temperatures and the displacements were recorded at different locations (S_j, $j=1,..,3$ for the displacements and T_i, $i=1,..,9$ for the temperatures). For real working tools, however, the number of temperature measurement locations are too high; thus we had to reduce this number without any loss of information. With the maximal correlation value of a chosen model we also have information about the quality of this ansatz. The higher the maximal correlation value the lower the unexplained variance in the model, i.e., the better the model. From the given nine temperature locations we can form $\Sigma_{i=1}^{9}\binom{9}{i}=511$ measurement combinations. If we had to calculate all these combinations for all measured data sets ($n=23$) and for all displacements we had to process 35 259 different models which would exceed our calculation capacity. Hence, we limited our study to maximal three different temperature locations, i.e., we are considering only $\Sigma_{i=1}^{3}\binom{9}{i}=129$ combinations (the maximal correlation for temperature triples was >0.99; i.e., we did not have to consider more than three temperature locations). For each model the maximal correlation was calculated for all measured data sets and all displacements. Then, we identified equivalent combinations for each displacement location S_j, $j=1,...,3$ separately. This was done in the following way: first the temperature combination with the cumulative highest value of the maximal correlation was determined (sum over all measurements), second all combinations were determined which show nearly the same results using the nonparametric Wilcoxon-test for paired samples. In this way we got 46 equivalent combinations for S_1, 7 for S_2 and 27 for S_3. Finally, we found only four different combinations which were optimal for all displacements: $\{T_1,T_2,T_8\}$, $\{T_2,T_4,T_8\}$, $\{T_2,T_5,T_8\}$, $\{T_2,T_8,T_9\}$. Note, the importance of the locations T_2 and T_8: both are included in all combinations. Without the information included in the third location, however, this tuple would be significantly worse. The last step was subjective: we decided to take the first combination $\{T_1,T_2,T_8\}$.

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