REMIND: The Equations

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This document describes the equations of the Integrated Assessment Model *REMIND* and represents a complement to the non-technical model description in Luderer et al. (2015). REMIND is developed as a modular framework of a multi-region energy-economy-climate model. The current framework provides a number of features that allows the representation of energy carriers and conversion technologies with various techno-economic characteristics. Moreover, the macroeconomic part contains a nested CES function that can have any structure. The regional models are solved as optimal growth models with equilibrium at the energy and capital markets. The present documentation introduces, jointly with the mathematical presentation of model equations, the *GAMS* notation of the model code. It gives an introduction to the abstract structure of the model and the modeling possibilities. The present documentation does not introduce the particular realization of a model version. Hence, the documentation opens up the possibility to implement individual model realizations.

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[*] Editorial responsibility

Contents

1 Preliminary remarks

1.1 Model Versions

The REMIND model comprises different model versions. The model is designed in a multi-regional structure. The versions differ with respect to the number of regions, module specifications and the climate module used to run climate policy scenarios.

This documentation covers the technical details of the major module specifications.

1.2 Notation convention

We use the following convention on notation:

- **Variables** are mostly written in a long form (to provide first intuition) starting with an upper case Latin letter.
- **Parameters** are represented by a single Greek letter. Exception: scenario parameters, initial values and boundary conditions on variables. They are denoted as the associated variable plus a particular marking (e.g.: K^0 is the initial value associated with the variable "capital" K). Another exception are parameters that are updated between iterations. They follow the notation of variables with an additional overhead bar.
- **Sets and Subsets** are written as upper case Latin letters plus single number. A single lower case latin letter is used for indices running over these sets. Indices (also the time and region index) are represented by comma separated subscripts.
- **Mappings** are written as M with single or multi-letter subscript. Mappings are used in GAMS to identify certain combinations of members of more than one set. The concept of mappings is explained in sec. 1.4.

Additional symbols denote special cases which may occur in any of the four types defined above:

- Temporal changes of items are symbolized by "Δ". (E.g.: ΔS is the change in the amount of a stockable quantity.) The time step length is symbolized by Δt.

1.3 Sets: The 'lattice' of the equations

Sets and subsets form the 'lattice' on which the equations are defined.

- t is the set of time steps from the initial point t_0 to the end point t_{end}.
- r is the set of regions.
- **Energy types** e**:** Various energy types like coal, electricity, natural gas for household use are defined and grouped into subsets according to

their characteristics (for example: primary, secondary, and final energy types p, s, f).

- **Technologies** c**:** This group covers all transformation technologies in the energy transformation or CCS chain. Again, there are subsets according to different characteristics.
- **Grade levels** g**:** Some items are characterized by different levels of quality.

1.4 Mappings: combining set elements

Mappings are used in GAMS to define combinations of set elements in order to avoid redundancy in the code. Consider the following example:

In the secondary to final energy transformation equation (cf. sec. 3.3.2, eq. 26), the variables "demand for secondary energy" ($DemSe$) and "production of final energy" ($ProdFe$) are indexed by time step, region, secondary energy type (s), final energy type (f), and transformation technology (c).

The equation is evaluated for all time steps and all regions ($\forall\ t, r$) and all defined combinations of secondary energy type, final energy type and technology ($\forall\ M_{s2f}$). The definition of the mapping contains the desired combinations $s \times f \times c$. This reduces the number of single equations generated in the compilation process, as "meaningless" combinations can be avoided.

Mappings can also be used in a summation index.

1.5 Equations and symbols used in the equations

The model equations are documented in the following chapters. The variables, parameters, sets/subsets and mappings are explained in tables at the end of each section sorted by these four groups. GAMS code notations are marked by a `special font`. The basic sets (r and t) and subsets named above in sec. 1.3 are not included in the tables again due to their high frequency of occurrence.

2 Economy module

2.1 The Intertemporal Social Welfare Function (`q_welfareGlob, q_welfare`)

The objective of the optimization is to maximize the total discounted intertemporal utility $\widetilde{U}_r$. It is calculated from consumption $C_{t,r}$ and population $P_{t,r}$ summing over all time steps taking into account the pure time preference rate ζ_r. Assuming an intertemporal elasticity of substitution of 1, it holds:

$$\widetilde{U}_r = \sum_t \left(\Delta t \cdot (1+\zeta_r)^{-t} \cdot \overline{P}_{t,r} \cdot \ln \left(\frac{C_{t,r}}{\overline{P}_{t,r}} \right) \right) \qquad \forall r \tag{1}$$

Global welfare sums up regional welfare U_r weighted by their Negishi weights W_r:

$$U = \sum_r \overline{W}_r \cdot \widetilde{U}_r \tag{2}$$

Allowing for an intertemporal elasticity of substitution η_r different from 1, it yields:

$$U = \sum_r \left(\overline{W}_r \cdot \sum_t \left(\Delta t \cdot (1+\zeta_r)^{-t} \cdot \overline{P}_{t,r} \cdot \frac{\left(\frac{C_{t,r}}{\overline{P}_{t,r}} \right)^{1-\eta_r} - 1}{1-\eta_r} \right) \right) \tag{3}$$

C	consumption	`vm_cons`
U	global welfare	`vm_welfareGlob`
$\widetilde{U}$	regional welfare	`v_welfare`
$\overline{P}$	population	`pm_pop`
$\overline{W}$	Negishi weight	`pm_w`
Δt	time step length	`pm_ts`
η	intertemporal elasticity of substitution	`p_ies`
ζ	time preference rate	`pm_prtp`

2.2 Budget equation (`qm_budget`)

Macroeconomic output Y - net of climate change damages (represented by damage factor Dam) - is added by imports of the final good ($MGood$), taking specific trade costs (τ^G) into account, which are assigned to the importer. The resulting output is used for consumption (C), for exports of the final good ($XGood$), for investments into the capital stock (I) [1], into R&D

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[1] Please note that the capital stock dynamics of the energy sector is treated separately in the energy system module. Associated investments enter the macroeconomic budget as investment costs $GInv$.

($InvRD$), and for the energy system cost components investments ($GInv$), fuel costs ($GFuel$) and operation & maintenance (GOM). Other additional costs like non-energy related greenhouse gas abatement costs ($GAbat$) and agricultural costs ($AgrC$), which are delivered by the land use model MAgPIE, are deduced from disposable output. Net tax revenues (Rev) (see section 2.9) and adjustment costs ($AdjNash$) (see section 5.2) converge to zero in the optimal solution(equilibrium point).

$$\begin{aligned} Y_{t,r} \cdot \ Dam_{t,r} + (1-\tau_r^G) \cdot MGood_{t,r} \ \geq \ & C_{t,r} + I_{t,r} + XGood_{t,r} \\ + \ & InvRD_{t,r} + GInv_{t,r} \\ + \ & GFuel_{t,r} + GOM_{t,r} \\ + \ & \sum_e \tau_{r,e}^M \cdot MRes_{t,r,e} \\ + \ & \sum_e \tau_{r,e}^X \cdot XRes_{t,r,e} \\ + \ & AdjInv_{t,r} + GPollut_{t,r} \\ + \ & Rev_{t,r} + AdjNash_{t,r} \\ + \ & \overline{GAbat_{t,r}} + \overline{AgrC_{t,r}} \\ \forall r, t \qquad & \end{aligned} \tag{4}$$

$AdjInv$	capital stock adjustment costs	`v_invMacroAdj`
$AdjNash$	adjustment costs of Nash algorithm	`vm_costAdjNash`
Dam	climate change damage factor	`vm_damage`
C	consumption	`vm_cons`
$GFuel$	fuel costs	`v_costFu`
$GInv$	investment costs	`v_costInv`
GOM	operation & maintenance costs	`v_costOM`
$GPollut$	pollution costs	`vm_costpollution`
I	investments into individual stocks of capital	`vm_invMacro`
$InvRD$	investments into R&D	`vm_invRD`
	investment into innovation	`vm_invInno`
	investment into immitation	`vm_invImi`
$MRes$	imports of primary energy	`vm_Mport(pety)`
$XRes$	exports of primary energy	`vm_Xport(pety)`
$MGood$	imports of the final good	`vm_Mport("good")`
Rev	net tax revenue	`vm_taxrev`
$XGood$	exports of the final good	`vm_Xport("good")`
Y	macroeconomic output	`vm_cesIO("inco")`
τ^X	additional domestic trade costs of primary energy export	`pm_costsTradePeFinancial`
τ^M	inter-regional trade costs of primary energy	`pm_costsTradePeFinancial`
τ^G	specific trade costs	`p_tradecostgood`
$\overline{GAbat}$	non-energy GHG abatement costs	`p_macCost`
$\overline{AgrC}$	agricultural production costs	`pm_ag_costs`
e	tradable energy type	`trade_pe`

2.3 The Production Function (`q_cesIO`)

The production function is a nested 'CES' (constant elasticity of substitution) production function. The macroeconomic output Y is generated by the inputs capital K, labor L, and total final energy E (as a macro-ecoomic aggregate in \$US units). The generation of total final energy is described by a CES production function as well, whose input factors are CES function outputs again. Hence, the outputs of CES nests are intermediates measured in \$US units. According to the Euler-equation the value of the intermediate equals the sum of expenditures for the inputs. Sector-specific final energy types represent the bottom end of the 'CES-tree'. These 'CES leaves' are measured in physical units and have a price in \$US per physical unit. The top of the tree is the total economic output Y measured in \$US.

In the code, you will find the generic form of the production function. It treats the various CES nests separately and the nests are inter-connetected via mappings. This equation calculates the amount of intermediate output in a time-step and region, $V_{r,t,o}$, from the associated factor input amounts $V_{r,t,i}$ according to:

$$V_{t,r,o} = \left(\sum_{M_{CES}} \xi_{r,i} \cdot (\theta_{r,t,i} V_{r,t,i})^{\rho_o} \right)^{\frac{1}{\rho_o}} \qquad \forall t, r, o \tag{5}$$

Parameter ρ_o is calculated from the elasticity of substitution, σ, according to the relation:

$$\sigma = \frac{1}{1 - \rho.}$$

Efficiency parameter $\theta_{r,t,i}$ is calculated as the product of an initial value and a scenario and time-dependent scaling factor. The mapping M_{CES} assigns the correct input types i to each output o.

All outputs (intermediate outputs and GDP) in the CES-tree represent monetary values. On the top of the CES-tree, macroeconomic output (GDP) is calculated from capital, labor, and total energy. If ρ_Y denotes the substitution elasticity, associated with GDP, Y, we thus have [2]:

$$Y = [\xi_K(\theta_K K)^{\rho_Y} + \xi_L(\theta_L L)^{\rho_Y} + \xi_E(\theta_E E)^{\rho_Y}]^{\frac{1}{\rho_Y}} \qquad \forall t, r \tag{6}$$

V	amount of production factor output	`vm_cesIO(in)`
Y	macroeconomic output	`vm_cesIO("inco")`
E	total final energy (as a production factor) in $US	`vm_cesIO("en")`
L	labor (equivalent to population)	`vm_cesIO("lab")`
K	capital	`vm_cesIO("kap")`
FE_j	final energy carrier j in physical units (e.g. EJ/yr)	`vm_cesIO(ppfen(in))`
θ	efficiency of input factors	`pm_cesdata("eff")`, `vm_effGr`
ρ	parameter, calculated from substitution elasticity σ	`pm_cesdata("rho")`
ξ	share parameter	`pm_cesdata("xi")`
i	input factors	`in`
o	production outputs	`out`
M_{CES}	combination of input types and associated output	`cesOut2cesIn(out,in)`

2.4 Capital stocks (`q_kapMo`,`q_kapMo0`)

Capital stock K is claculated recursively. Its amount in the previous time step is devaluated by an annual depreciation factor δ_k and enlarged by investments I. Both depreciation and investments are expressed as annual values, so the time step length Δt is taken into account.

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[2] For clarity, the regional indices and time arguments of Y, K, L, E and the associated efficiency parameters have been dropped here.

$$K_{t+1,r} = K_{t,r}\left(1 - \Delta t \cdot \delta\right) + \Delta t \cdot\ I_{r,t} \qquad \forall t, r \tag{7}$$

Initial values are assigned from exogenous data K^0:

$$K_{t,r} = K^0_r \qquad \forall r, t = 0 \tag{8}$$

I	investments	`vm_invMacro`
K	macroeconomic capital stock	`vm_cesIO("kap")`
K^0	Initial value of capital stock	`pm_cesdata("kap")`
δ	annual depreciation factor	`p_delta_kap`
Δt	time step length	`pm_ts`

2.5 Labor (`q_balLab`)

The labor available in every time step and every region, $L_{r,t}$, comes from exogenous data. It is the population corrected by the population age structure, which results in the labour force of people agged 15 to 65. The labor participation rate is not factored into the labour supply (as it would only imply a rescaling of parameters without consequences for the model's dynamic). The labour market balance equation reads as follows:

$$L_{t,r} = \overline{L}_{t,r} \qquad \forall t, r \tag{9}$$

L	labor available in every time step and region	`v_cesIO("lab")`
$\overline{L}$	exogenous data for available labor	`p_p_lab`

2.6 Final Energy balance (`q_balfE`, `q_esm2macro`, `q_transFE2es`)

The final energy balance equals the production of final energy P_f of type f in time-step t and region r to its demand as an input factor of the production function $V_f^r(t)$. Both variables are measured in energy units.

$$V_{t,r,f} = P_{t,r,f} \qquad \forall t, r, f \tag{10}$$

P	final energy production	`vm_prodFE`
V	final energy demand, production factor	`v_demFE`, `v_cesIO`
f	final energy type	`fety,ppfen`

2.7 Trade balances (`q80_tradebal`)

In each time step, exports and imports of each tradeable entity are globally balanced. This applies for exports and imports of each energy type e (oil, gas, coal, biomass, uranium), final good, and emission permits. The way of getting international markets to be cleared is different for both major solution concepts (see section 5). With the Negishi solution approach, the following trade balance is formulated explicitly:

$$\sum_r (X_{t,r,j} - M_{t,r,j}) = 0 \qquad \forall\ t, j \tag{11}$$

M_j	import of commodity j	`vm_Mport(trade)`
	energy imports	`vm_Mport(trade_pe)`
	import of final goods	`vm_Mport("good")`
	permit imports	`vm_Mport("perm")`
X_j	export of commodity j	`vm_Xport(trade)`
	energy exports	`vm_Xport(trade_pe)`
	export of final goods	`vm_Xport("good")`
	permit exports	`vm_Xport("perm")`
j	all tradeable goods	`trade`

2.8 Emissions permit allocation (`q41_perm_alloc_cap`)

Emission permit allocation (QP) is either derived from a predefined emission cap ($EmCap$)

$$QP_{t,r} = \theta_{t,r} \cdot \overline{EmCap}_t \quad \forall\ t, r \tag{12}$$

or based on an endogenous GHG emission path Q_{tot}

$$QP_{t,r} = \theta_{t,r} \cdot Q_t \quad \forall\ t, r \tag{13}$$

To calculate the regional shares (θ) on the global permit budget, three different allocation scenarios are possible:

Contraction and Convergence

(Negishi mode only)

$$\theta_{t,r} = \left(\lambda_t \cdot \frac{\overline{L}_{t,r}}{\sum_r \overline{L}_{t,r}} + (1 - \lambda_t) \cdot \frac{Q_r^0}{\sum_r Q_r^0} \right) \quad \forall\ t, r \tag{14}$$

According to this rule, permit allocation converges from status quo towards an equal per capita allocation. The convergence parameter λ increases linearly from zero at the beginning of the time horizon (2005) to 1 at the convergence time (2050).

GDP intensity

(Negishi mode only)

$$\theta_{t,r} = \frac{\overline{Y}_{t,r}}{\sum_r \overline{Y}_{t,r}} \qquad \forall\ t, r \tag{15}$$

Permits are allocated among regions in proportion to their share on global GDP in the baseline scenario.

Population share

$$\theta_{t,r} = \frac{\sum_t \overline{L}_{t,r}}{\sum_t \sum_r \overline{L}_{t,r}} \qquad \forall\ t, r \tag{16}$$

Permits are allocated among regions (at each time step) in proportion to their population share over the entire time horizon.

Q	global GHG emissions	`vm_co2eqGlob`
QP	allocated emission permits	`vm_perm`
$\overline{EmCap}$	emission cap	`pm_emicapglob`
$\overline{L}$	population scenario	`pm_datapop`
Q^0	GHG emissions in 2005	
θ	regional share on global permit budget	`pm_shPerm`
$\overline{Y}$	GDP in the business-as-usual scenario	`pm_gdp_bau`
λ	convergence parameter	`p41_lambda`

2.9 Tax mechanism (`q21_taxrev`)

Taxes and subsidies are implemented in a budget-neutral way: In each solution iteration i, the net revenue $Rev(r,t,i)$ is the difference between the paid taxes and the revenue of the previous iteration (Equation 17). This way, the net revenue that is a summand in the budget equation (see section 2.2) converges iteratively to zero. Therefore the taxes do not have an effect on the available budget, but the optimization is subject to the distorting marginal effect of taxes and subsidies.

$$Rev_{r,t,i} = \left(\sum_f \tau_{r,t,f,i} \cdot Act_{r,t,f,i} \right) - Rev_{r,t,i-1} \tag{17}$$

Taxes (and subsidies) for the following factors f are implemented, although tax rates are set to zero in default scenario settings with the exception of final energy taxes/subsidies: greenhouse gas emissions, final energy use, primary-to-secondary energy technologies, resource export, SO_2 emissions, bioenergy use. For some of them, the tax rate τ_t can itself be a function of the activity Act_t or other variables.

Rev	net tax revenue (converges to zero)	`vm_taxrev`
τ	tax rate	`pm_tau_*`
Act	activity level for taxed variables	

3 Energy System Module

3.1 Energy system costs

3.1.1 Fuel costs (`q_costfu`)

Fuel costs are associated with the use of exhaustible primary energy (fossils, uranium) and biomass. In the latter case, resources are divided into several grades, and each grade has fixed specific costs. In the former case, specific fuel costs are a function of previous cumulative extraction (**"Rogner-curve"**).

$$
\begin{aligned}
CostFu_{t,r} = &\sum_{(b,g)} \left(\tau_{t,r,b,g} \cdot FuelEx_{t,r,b,g}\right) \\
&+ \sum_{e} \left(\chi_{r,e} + \psi_{r,e} \left(\frac{\sum_t \Delta t \cdot FuelEx_{t,r,e,g}}{\phi_{r,e}} \right)^{\zeta_{r,e}} FuelEx_{t,r,e} \right) \\
&\qquad \forall\, t, r \quad \forall (e,g) \in M_{b2g}
\end{aligned}
\tag{18}
$$

$FuelEx$	fuel extraction of primary energy b or e	`vm_fuEx`
$CostFu$	overall fuel costs	`v_costFu`
τ	cost per unit of fuel b with grade level g	`dataperen("cost")`
χ, ψ, ϕ, ζ	parameters to characterize the exhaustible fuel cost curve	`datarog`
b	biomass energy tpyes	
e	exhaustible primary energy types	`peExPol`
M_{b2g}	combinations of primary energy types and grade levels (covers only biomass)	`peren2rlf`

3.1.2 Investment Costs (`q_costInv`)

Specific investment costs of learning technologies are a model-endogenous variable; those of non-learning technologies are fixed to constant values. Total investment costs *CostIn* are the product of specific costs and capacity additions *CapAdd* plus adjustment costs *AdjCost*:

$$
CostIn_{t,r} = \sum_{c} \left(J_{t,r,c} \cdot \sum_{(c,g)} CapAdd_{t,r,c,g} + AdjCost_{t,r,c} \right) \qquad \forall\, t, r \;\; \forall (c,g) \in M_{c2g} \tag{19}
$$

$$
AdjCost_{t,r} = \sum_{c} \left(J_{t,r,c} \cdot \beta_c \cdot \frac{\left(\frac{CapAdd_{t,r,c} - CapAdd_{t-1,r,c}}{\Delta t} \right)^2}{CapAdd_{t-1,r,c} + \kappa_{t,r} \cdot \pi_c} \right) \qquad \forall\, t, r \;\; \forall (c,g) \in M_{c2g} \tag{20}
$$

$CostIn$	investment costs	`v_costInv`
J	specific investment costs per unit of capacity addition of a learning technology l	`vm_costTeCapital`
$CapAdd$	addition to the capacity of technology c and l of grade level g	`vm_deltaCap`
$AdjCost$	Adjustment costs	`v_adjFactor`
β	adjustment cost coefficient	`p_adj_coeff`
κ	build-up capacity	`p_adj_seed_reg`
π	multiplicative factor	`p_adj_seed_te`
c	all technologies	`en2en` `teNoTransform`
M_{c2g}	combination of technologies and grade levels	`te2rlf`

3.1.3 Operation and Maintenance Costs (`q_costom`)

O&M costs result from

- maintenance of existing facilities according to their capacity (**fixed O&M costs**) and
- operation of energy transformations according to the amount of produced secondary and final energy (**variable O&M costs**).

Addition of both contributions yields total O&M costs $CostOM$:

$$\begin{aligned} CostOM_r = \sum_{(p,s,f,c)} \Bigg(& \mu_{r,c} \sum_{(c,g)} \Big(J_{r,c} \cdot Z_{r,c,g} \Big) \\ & + \rho_{r,c} \cdot \Big(ProdSe_{r,p,s,c} + ProdFe_{r,s,f,c} \Big) \Bigg) \end{aligned} \tag{21}$$

$$\forall (p,s,f,c) \in M_{e2\tilde{e}}, \qquad \forall (c,g) \in M_{c2g} \quad \forall (l,g) \in M_{l2g}$$

$CostOM$	operation & maintenance costs	`v_costOM`
J	specific investment costs for adding capacity of technology c	`vm_costTeCapital`
$ProdSe$	production of secondary energy	`v_prodSe`
$ProdFe$	production of final energy	`v_prodFe`
Z	capacity of technology c and l	`vm_cap`
μ	fixed specific O&M costs	`data("omf")`
ρ	variable specific O&M costs	`data("omv")`
c	all technologies	`en2en` `teNoTransform`
$M_{e2\tilde{e}}$	definition of general energy transformation	`temapall`
M_{c2g}, M_{l2g}	combination of technologies and grade levels	`te2rlf`

3.2 Energy Balance Equations

Energy balance equations equate the production P of and demand D for each primary, secondary and final energy; so the general structure is:

$$\sum_{all} Prod_{t,r,e} = \sum_{all} Dem_{t,r,e} \quad \forall\ t, r \quad \forall \text{ energy types e}$$

where "all" means all possible ways of energy transformation that produce or demand energy type e.

3.2.1 Primary Energy Balance (`q_balPe`)

Supply of primary energy $ProdPe$ equals demand on Primary energy $DemPe$.

$$\sum_{(p,g)\in M_{p2g}} ProdPe_{t,r,p,g} = \sum_{(p,s,c)\in M_{p2s}} DemPe_{t,r,p,s,c} \quad \forall\ t, r \ \ \forall\ p \qquad (22)$$

$DemPe$	demand for primary energy	`vm_demPe`
$ProdPe$	production of primary energy	`vm_prodPe`
c	all technologies	`te`
p	all primary energy types	`entyPe`
s	all secondary energy types	`entySe`
g	all resource grades	`rlf`
M_{p2g}	combination of primary energy types with grade levels	`enty2rlf2`
M_{p2s}	combination of primary and secondary energies with their conversion technology	`pe2se`

3.2.2 Secondary Energy Balance (`q_balSe`)

The secondary energy balance comprises the following terms:

- Secondary energy can be produced ($ProdSe$) from primary or (another type of) secondary energy.
- Own consumption of secondary energy occurs from the production of secondary and final energy, and from CCS technologies. Own consumption is calculated as the product of the respective production ($ProdSe$, $ProdFe$, or CCS as the amount of CO_2 in the respective CCS chain step) and a negative coefficient ξ. Mapping M_{oc} defines possible combinations: the first two enty types of the mapping define the underlying transformation process, the 3rd argument the technology, and the 4th argument specifies the consumed energy type.
- Couple production is modeled as own consumption, but with a positive ξ.

- Secondary energy can be demanded (*DemSe*) to produce final or (another type of) secondary energy.

$$
\begin{aligned}
&\sum_{(p,s,c)\in M_{p2s}} ProdSe_{t,r,p,s,c} + \sum_{(\tilde{s},s,c)\in M_{s2s}} ProdSe_{t,r,\tilde{s},s,c} \\
&+ \sum_{(e,\tilde{s},c,s)\in M_{oc}} \xi_c \cdot ProdSe_{t,r,e,\tilde{s},c} + \sum_{(e,f,c,s)\in M_{oc}} \xi_c \cdot ProdFe_{t,r,e,f,c} \\
&+ \sum_{(e,\tilde{e},c,s)\in M_{oc}} \xi_c \cdot CCS_{t,r,e,\tilde{e},c} \qquad\qquad \text{if} \quad \text{c} \in \text{T1} \\
&= \\
&\sum_{(s,f,c)\in M_{s2f}} DemSe_{t,r,s,f,c} + \sum_{(s,\tilde{s},c)\in M_{s2s}} DemSe_{t,r,s,\tilde{s},c} \\
&+ \sum_{c\in T2} StorL_{t,r,c} \qquad\qquad \text{if} \quad \text{s = electricity}
\end{aligned}
\tag{23}
$$

$$\forall\, t,r \quad \forall s$$

$DemSe$	demand for secondary energy	`v_demSe`
$ProdSe$	production of secondary energy	`vm_prodSe`
$ProdFe$	production of secondary energy	`vm_prodFe`
CCS	CCS CO_2 emissions	`v_co2CCS`
$ImpSe$	Import of secondary energy	`v_MpSE`
$ExpSe$	Export of secondary energy	`v_XpSE`
$StorL$	Storage losses of electricity	`v_storloss`
ξ	own consumption factor c	`p_dataoc`
c	all technologies	`te`
e	all energy types	`enty`
p	all primary energy types	`entyPe`
s	all secondary energy types	`entySe`
f	all final energy types	`entyFe`
$T1$	technologies used to deposit CO_2 underground	`teccs2rlf(te,rlf)`
$T2$	VRE technologies producing electricity (wind, PV, CSP)	`teVRE`
M_{p2s}	combination of primary and secondary energies with their conversion technology	`pe2se`
M_{s2s}	combination of secondary and secondary energies with their conversion technology	`se2se`
M_{s2f}	combination of secondary and final energies with their conversion technology	`se2fe`
M_{oc}	combination of energy types and technologies that have couple production or several inputs	`oc2te`

3.3 Energy Transformation Equations

Taking the technology-specific transformation efficiency η into account, the equations describe the transformation of an energy type to another type; note that energy type e entering a transformation is *demanded* (Dem), the resulting energy type $\tilde{e}$ is *produced* ($Prod$):

$$\eta_{t,r,c} \cdot Dem_{t,r,e,c} = Prod_{t,r,\tilde{e},c} \quad \forall\, t, r \quad \forall (e, \tilde{e}, c) \in M_{e2\tilde{e}} \tag{24}$$

and the allowed combinations of e and $\tilde{e}$ are primary to secondary, secondary to secondary, secondary to final energy, and final energy to energy services.

3.3.1 Primary Energy to Secondary Energy (`q_transPe2Se`)

Depending on the detail of the technology representation, the transformation technology's efficiency (η_t) can depend either only on the current year or on the year when a specific technology was built; in the latter case, the production ($ProdSe$) is replaced by the equivalent product of the depreciated capacity additions ($\omega \cdot CapAdd$) and load factor (ν) to assign the η_t value valid at the year of the capacity addition (compare with sections 3.5.3 and 3.5.1):

$$DemPe_{t,r,p,s,c} = \begin{cases} \dfrac{1}{\eta_{t,r,c}} \cdot ProdSe_{t,r,p,s,c} & \text{if} \quad \mathrm{c} \in \mathrm{T1} \\ (1 - ERet_{t,r,c}) \cdot \nu_{r,c} \cdot \sum\limits_{\tilde{t} \le t} \Delta\tilde{t} \cdot \dfrac{\omega_{t-\tilde{t},r,c} \cdot CapAdd_{\tilde{t},r,c}}{\eta_{\tilde{t},r,c}} & \text{if} \quad \mathrm{c} \in \mathrm{T2} \end{cases} \tag{25}$$

$$\forall\, t, r \quad \forall (p, s, c) \in M_{p2s}$$

$DemPe$	demand for primary energy	`vm_demPe`
$ProdSe$	production of secondary energy	`vm_prodSe`
$CapAdd$	addition to the capacity of technology c	`vm_deltaCap`
$ERet$	Share of capacities that is retired early	`v_capEarlyReti`
η	efficiency of technology c	`pm_eta_conv, pm_dataeta`
ν	load factor of technology c	`pm_cf`
ω	depreciation factor reflecting reduced capacity use as technology ages	`pm_omeg`
Δt	time step length of each time step	`pm_ts`
c	all technologies	`te`
$T1$	technologies where the efficiency only depends on the current year (simplified assumption)	`teEtaConst`
$T2$	technologies where the efficiency is determined by the build year and is tracked throughout time	`teEtaIncr`
M_{p2s}	combination of primary and secondary energies with their conversion technology	`pe2se`

3.3.2 Secondary Energy to Final Energy (`q_se2fetrans`)

$$DemSe_{t,r,s,f,c} = \frac{ProdFe_{t,r,s,f,c}}{\eta_{t,r,c}} \tag{26}$$

$$\forall\ t, r \quad \forall(s, f, c) \in M_{s2f}$$

3.3.3 Secondary Energy to Secondary Energy (`q_se2setrans`)

$$DemSe_{t,r,s,\tilde{s},c} = \frac{ProdSe_{t,r,s,\tilde{s},c}}{\eta_{t,r,c}} \tag{27}$$

$$\forall\ t, r \quad \forall(s, \tilde{s}, c) \in M_{s2s}$$

3.3.4 Final Energy to Energy Services (`q_fe2estrans`)

$$DemFe_{t,r,f,e,c} = \frac{ProdEs_{t,r,f,e,c}}{\eta_{t,r,c}} \tag{28}$$

$$\forall\ t, r \quad \forall(f, e, c) \in M_{f2e}$$

DemSe	demand for secondary energy	`v_demSe`
DemFe	demand for final energy	`v_demFe`
ProdSe	production of secondary energy	`vm_prodSe`
ProdFe	production of final energy	`vm_prodFe`
ProdEs	production of energy services	`vm_prodEs`
η	efficiency of technology c	`pm_eta_conv`
c	all technologies	`te`
M_{s2f}	combination of secondary and final energies with their conversion technology	`se2fe`
M_{s2s}	combination of secondary and secondary energies with their conversion technology	`se2se`
M_{f2e}	combination of secondary and secondary energies with their conversion technology	`fe2es`

3.4 Treatment of VRE technologies (storage & grid requirements, resource competition)

- Each variable renewable technology c_{VRE} (VRE: wind, PV, CSP) requires a certain amount of its respective storage technology (stor: storwind, storPV, storCSP). Total storage needs from c_{VRE} increase with the absolute amount of electricity generated by c_{VRE} as well as with its share in total electricity generation.
- The specific relative storage requirement (StorSh) increases with increasing share of the respective VRE technology (ShSeel) as well as with increasing share of all VRE technologies that use a resource with similar fluctuations (M_{link}) (reduced by a factor 1/3 to represent only partial correlation between fluctuations). See eq. 33.
- Total storage capacity needed for a VRE technology c_{VRE} is calculated from the storage losses (StorL). See eq. 41.
- Total storage losses (StorL) are given by round-trip conversion losses multiplied by the product of specific relative storage requirement (StorSh) with the total usable electricity production of c_{VRE} . See eq. 32.
- Round-trip conversion losses as percent of output are given by $(1-\eta)/\eta$, with η the conversion efficiency of the storage technology
- Total usable electricity production of c_{VRE} is equal to the total seel production of c_{VRE} minus the losses of the associated storage technology. See eq. 30.

3.4.1 Usable electricity

Share of technology te in total electricity production (`q_shSeEl`)

The share of electricity generation of one technology is calculated by dividing the useful secondary energy electricity (seel) production of one VRE technology by the combined usable seel production from primary energy and couple production. As storage losses are not used for anything, they are not counted in the 'usable seel' variables (else the share of a VRE technology might be larger than 100 percent).

$$ShSeel_{t,r,\mathbf{s},\mathbf{c}} = 100 \cdot \frac{UseSeTe_{t,r,\mathbf{s},\mathbf{c}}}{UseSe_{t,r,\mathbf{s}}} \tag{29}$$

$$\forall\ t, r \quad \forall \mathbf{c} \in T1 \quad \mathbf{s} = \text{electricity}$$

Usable electricity from VRE technology c (`q_usableSeTe`)

$$UseSeTe_{t,r,\mathbf{s},\mathbf{c}} = \left(\sum_{(p,\mathbf{s},\mathbf{c}) \in M_{p2s}} ProdSe_{t,r,p,\mathbf{s},\mathbf{c}} \right) - StorL_{t,r,\mathbf{c}} \tag{30}$$

$$\forall\ t, r \quad \forall \mathbf{c} \in T1 \quad \mathbf{s} = \text{electricity}$$

Total usable electricity from primary energy and couple production (`q_usableSe`)

$$\begin{aligned} & UseSe_{t,r,\mathbf{s}} \\ & = \\ & \sum_{(p,\mathbf{s},c) \in M_{p2s}} ProdSe_{t,r,p,\mathbf{s},c} \\ & + \sum_{(p,\tilde{s},c,\mathbf{s}) \in M_{oc}} \xi_c \cdot ProdSe_{t,r,p,\tilde{s},c} && \text{if} \quad \xi_c > 0 \\ & - StorL_{t,r,\mathbf{c}} \end{aligned} \tag{31}$$

$$\forall\ t, r \quad \mathbf{s} = \text{electricity}$$

ShSeel	Share of net seel production from techn. c in total net seel production	`v_shSeEl`
UseSeTe	usable electricity from technology c	`v_usableSeTe`
UseSe	total usable electricity	`v_usableSe`
ProdSe	production of secondary energy	`vm_prodSe`
StorL	Storage losses of electricity	`v_storloss`
ξ	own consumption factor c	`pm_prodCouple`
c	all technologies	`te`
p	all primary energy types	`pety(enty)`
s	all secondary energy types	`sety(enty)`
$T1$	VRE technologies producing electricity (wind, PV, CSP)	`teVRE(te)`
M_{p2s}	combination of primary and secondary energies with their conversion technology	`pe2se`
M_{oc}	combination of energy types and technologies that have couple production or several inputs	`pc2te`

3.4.2 Calculate storage losses and requirements

Storage losses (`q_storloss`)

$$StorL_{t,r,c} = \frac{StorSh_{t,r,c}}{100} \cdot UseSeTe_{t,r,c} \cdot \sum_{(c,\tilde{c}) \in M_{stor}} \frac{1-\eta_{\tilde{c}}}{\eta_{\tilde{c}}} \tag{32}$$

$$\forall\ t,r \quad \forall\ c \in \mathrm{T1}$$

Storage requirements (`q_shStor`)

$$StorSh_{t,r,c} > 100 \cdot \lambda_{r,c} \cdot \left(\frac{1}{100} \cdot ShSeel_{t,r,c} + \frac{\sum_{(c,\tilde{c}) \in M_{link}} ShSeel_{t,r,\tilde{c}}}{3} \right)^{\mu_{r,c}} \tag{33}$$

$$\forall\ t,r \quad \forall\ c \in \mathrm{T1}$$

UseSeTe	usable electricity from technology c	`v_usableSeTe`
StorSh	auxiliary variable: the relative need for storage	`v_shStor`
ShSeel	Share of net electricity production from technology c in total net electricity production	`v_shSeEl`
η	conversion efficiency of the storage	`pm_eta_conv`
λ	factor for up/downscaling requirements in different regions/for different technologies (default: 1)	`p_stor_factor`
μ	exponent determining how fast marginal storage requirements increase with VRE share (default: 1)	`p_storexp`
c	all technologies	`te`
$T1$	VRE technologies producing electricity (wind, PV, CSP)	`teVRE(te)`
M_{stor}	combination of VRE technologies and storage technologies	`VRE2teStor`
M_{link}	combination of VRE technologies that use the same resource and thus are correlated	`VRE2teVRElinked`

3.4.3 Required long-distance transmission grid

The variable renewable technologies c_{VRE}, for which the potential is usually concentrated in few regions of a continent, require explicit transmission grid expansion on top of the linear grid costs required for all electricity use.

Calculating the grid capacity requirements (`q_limitCapTeGrid`)

$$CapGrid_{t,r} = \sigma_r \sum_{c \in T1} \zeta_c \cdot \; ProdSe_{t,r,c} \tag{34}$$

$$\forall \; t, r$$

CapGrid	Additional grid capacity for VRE technologies	`vm_cap`
ProdSe	usable electricity from technology c	`vm_ProdSe`
σ	factor for up/downscaling grid requirements in different regions (default: 1)	`p_grid_factor`
ζ	factor for up/downscaling grid requirements for different technologies	
c	all technologies	`te`
$T1$	VRE technologies producing electricity (wind, PV, CSP)	`tegrid(te)`

3.4.4 Competition for resources for renewable energies (`q_limitGeopot`)

Several renewable technologies use the same resource potential, e.g., CSP and PV both compete for solar irradiance and thus cannot use the same piece of land to generate electricity. The model needs to make sure that the sum of area used by both technologies is smaller than the total usable area/technical potential.

$$\gamma_{r,p,g} > \sum_{(p,c,g) \in M_{p2cg}} \frac{CapDist_{t,r,c,g}}{\iota_{r,c}} \qquad \forall \; t, r, g \;\; \forall \; p \in \mathrm{P1} \tag{35}$$

$CapDist$	Capacity of renewable technology distributed to different resource grades	`v_capDistr`
γ ι	maximum technical potential capacity per area of a renewable technology	`p_dataplot("q_limitGeopot")` `pm_data("luse")`

c
p
g
	all technologies
	primary energy types
	all resource grades
		`te`
		`pety`
		`rlf`

$P1$	all renewable PE with resource competition from different technologies	`peReComp`
M_{p2cg}	mapping of renewable primary energies with resource competition to technologies and resource grades	`teReComp2pe`

Final Energy Balance

The final energy balance is described in the economy module. See section 2.6.

3.5 Capacities

The following equations are at the core of the energy system, by linking the energy flows ($ProdXX$) with required capacities Cap of the corresponding conversion technologies. The equations are structurally equivalent throughout the chain from primary energy to secondary to final energy and lastly to energy services, and similar equations also apply to the CCS chain and storage.

3.5.1 Capacity constraints for energy transformations

Capacity constraints for primary to secondary energy transformation (`q_limitCapSe`)

$$ProdSe_{t,r,p,s,c} = \sum_{(c,g)\in M_{c2g}} \nu_{r,c} \cdot \nu^{*}_{r,c,g} \cdot Cap_{t,r,c,g} \quad \forall\, t,r \;\; \forall\, (p,s,c) \in M_{p2s} \tag{36}$$

Capacity constraints for secondary to secondary energy transformation (`q_limitCapSe2se`)

$$ProdSe_{t,r,s,\tilde{s},c} = \sum_{(c,g)\in M_{c2g}} \nu_{r,c} \cdot \nu^{*}_{r,c,g} \cdot Cap_{t,r,c,g} \quad \forall\, t,r \;\; \forall\, (s,\tilde{s},c) \in M_{s2s} \tag{37}$$

Capacity constraints for secondary to final energy transformation (`q_limitCapFe`)

$$ProdFe_{t,r,s,f,c} = \sum_{(c,g)\in M_{c2g}} \nu_{r,c} \cdot \; Cap_{t,r,c,g} \quad \forall\, t,r \;\; \forall\, (s,f,c) \in M_{s2f} \tag{38}$$

Capacity constraints for final energy to energy service transformation (`q_limitCapEs`)

$$ProdEs_{t,r,f,e,c} = \sum_{(c,g)\in M_{c2g}} \nu_{r,c} \cdot \; Cap_{t,r,c,g} \quad \forall\, t,r \;\; \forall\, (f,e,c) \in M_{f2e} \tag{39}$$

Constraint on the Share of Electricity from CHP (`q_limitCapTeChp`)

$$\sum_{(p,s,c)\in M_{CHP}} ProdSe_{t,r,p,s,c} \leq \phi_r \cdot \sum_{(p,s,c)\in M_{p2s}} ProdSe_{t,r,p,s,c} \qquad \forall t,r \quad \text{if} \quad \text{s} = \text{electricity} \tag{40}$$

Capacity constraints for storage (`q_limitCapTeStor`)

$$\sum_{(c,\tilde{c})\in M_{stor}} StorL_{t,r,\tilde{c}} \cdot \frac{\eta_{t,r,c}}{1-\eta_{t,r,c}} = \sum_{(c,g)\in M_{c2g}} \nu_{r,c} \cdot \nu^{*}_{r,c,g} \cdot Cap_{t,r,c,g} \quad \forall\, t,r \;\; \forall\, c \in T1 \tag{41}$$

$ProdSE$	production of secondary energy	`vm_prodSe`
$ProdFE$	production of final energy	`vm_prodFe`
$ProdES$	production of energy service	`vm_esprod`
Cap	capacity of technology c	`vm_cap`
$StorL$	energy loss due to storage process	`v_storloss`
ν	capacity factor associated with technology c	`pm_cf`
ν_g^*	scaling of the load factor ν dependent on grade level g	`pm_dataren("nur")`
ϕ	maximum share of electricity from CHP on overall electricity generation	`p_shCHP("bscu")`
η	conversion efficiency of the storage	`pm_eta_conv`
c	all technologies	`te`
e	all energy types	`enty`
p	all primary energy types	`pety(enty)`
s	all secondary energy types	`sety(enty)`
f	all final energy types	`fety(enty)`
$T1$	storage technologies (storwind, storPV, storCSP)	`teVRE`
M_{p2s}	primary to secondary energy transformation technologies	`pe2se`
M_{s2s}	secondary to secondary energy transformation technologies	`se2se`
M_{s2f}	secondary to final energy transformation technologies	`se2fe`
M_{f2e}	final energy to energy service transformation technologies	`fe2es`
M_{c2g}	combination of energy technologies and grade levels	`teall2rlf`
M_{stor}	combination of VRE technology and its respective storage technology	`VRE2teStor`

3.5.2 Capacity constraints for CCS technologies (`q_limitCapCCS`)

$$R_{t,r,i,i+1,c,g} = \sum_{(c,g)\in M_{c2g}} \nu_{r,c} \cdot Cap_{t,r,c,g} \quad \forall\, t,r \;\; \forall\, (i,i+1,c) \in M_{CCS} \qquad (42)$$

R	amount of CO_2 in step i of the CCS chain to be transformed to the next one using technology c with grade level g	`v_co2CCS`
Cap	capacity of CCS transformation technology c with grade level g	`vm_cap`
ν	capacity factor associated with technology c	`pm_cf`
c	all technologies	`te`
i	step of the CCS process chain	
M_{CCS}	definition of CCS steps and associated technologies	`ccs2te`
M_{c2g}	combination of technology and grade levels	`teCCS2rlf`

3.5.3 Capacity Depreciation (`q_cap`)

The capacities of vintaged technologies (c_{vin}) depreciate according to a vintage depreciation scheme, with generally low depreciation at the beginning of the lifetime, and fast depreciation around the average lifetime. Depreciation can generally be tracked for each grade separately. By implementation, however, only grades of level 1 are affected. The depreciation of any fossil technology can be accelerated by early retirement ($ERet$), which is a crucial way to quickly phase out emissions after the implementation of stringent climate policies.

$$Cap_{t,r,c,g} = (1 - \ ERet_{t,r,c}) \cdot \left(\sum_{(c,\tilde{t}) \in M_{tl}} \Delta t \cdot \omega_{r,\tilde{t},c} \cdot \ CapAdd_{t-\tilde{t},r,c,g} \right)$$

$$\forall \ t, r \ \ \forall \ (c, g) \in M_{c2g} \quad (43)$$

Cap	capacity of technology c	`vm_cap`
$CapAdd$	addition of capacity	`vm_deltaCap`
$ERet$	share of prematurely retired capacities	`v_capEarlyReti`
ω	weight factor of addition to technology c's capacity prior to initial time	`pm_omeg`
Cap^0	initial capacity of technology c	`data("cap0")`
$\tilde{t}$	life time	`opTimeYr`
c	technologies	`te`
M_{c2g}	combination of technologies and grade levels	`te2rlf`
M_{tl}	set of possible combinations of vintage technologies and life time indices	`opTimeYr2te`

3.6 Learning equation (`q_costTeCapital`, `qm_deltaCapCumNet`)

Technological change is an important driver of the evolution of energy systems. For mature technologies, such as coal-fired power plants, the evolution of techno-economic parameters is prescribed exogenously. For less mature technologies with substantial potential for cost decreases via learning-by-doing, investment costs are determined via an endogenous one-factor learning curve approach that assumes floor costs θ_c:

$$IC_{t,c} = \alpha_c \cdot \sum_r CCap_{t,r,c}^{\beta_c} + \theta_c \quad \forall\, t,r\ , \quad \forall\, c \in \text{ T1}$$

with

$$\alpha_c = \frac{IC_c^0 - \theta_c}{\sum_r CCap_{r,c}^{0\ \beta_c}} \quad \forall\, r\ ; \quad \forall\, c \in \text{T1}$$

and β_c calculated from the learning rate $\tilde{\beta}$ (relative cost decrease when cumulated capacities double):

$$\beta_c = \frac{\ln(1 - \tilde{\beta}_c)}{\ln 2} \quad \forall\, r\ ; \quad \forall\, c \in \text{T1}$$

It should be noted that the learning rate in this formula applies to the learning part only - it is thus larger than the empirically observed learning rate if $\theta > 0$. The cumulated capacities $CCap$ are calculated as

$$CCap_{t+1,r,c} = CCap_{t,r,c} + \Delta t \cdot CapAdd_{t,r,c} \quad \forall\, t,r; \ \ \forall\, c \in \text{ T1};$$

This is equivalent to the common formulation of learning curves in the literature

$$IC_{t,c} = \tilde{\alpha}_c \cdot \left(\frac{\sum_r CCap_{t,r,c}}{\sum_r CCap_{t,r,c}^0} \right)^{\beta_c} + \theta_c \quad \forall\, t,r\ ; \quad \forall\, c \in \text{T1}$$

where $\tilde{\alpha}_c$ represents the difference between initial costs and floor costs.

IC	specific investment costs for adding capacity of learning technology c	`vm_costTeCapital`
$CCap$	cumulated capacity of technology c	`vm_capCum`
θ	floor costs of learning technology c	`pm_data("floorcost")`
α	parameter of learning technology c	`pm_data("learnMult_wFC")`
β	parameter of learning technology c	`pm_data("learnExp_wFC")`
$\tilde{\alpha}$	difference between initial costs and floor costs	
$\tilde{\beta}$	learning rate	
T1	set of technologies subject to endogenous learning	`teLearn`

3.7 Resource and Potential Constraints

3.7.1 Fuel extraction (`qm_fuel2pe`)

To ensure that energy demand matches production, a balance equation links primary energy production to exhaustible resources extraction. More specifically primary energy production $ProdPe$ must equal the extraction $FuelEx$ and the traded exhaustible primary energy carriers e (i.e. coal, oil, gas, uranium and biomass). Trade is designed so that each model region can import ($MRes$) or export ($XRes$) any amount of tradable primary energy. It is important to note that specific trade costs τ are taken into account.[3]

$$\begin{aligned} ProdPe_{t,r,e,g} = & \sum_{M_{e2g}} FuelEx_{t,r,e,g} \\ & - (XRes_{t,r,e} - (1 - \tau_{r,e}) \cdot MRes_{t,r,e}) \qquad \text{if} \quad e \in E1 \\ & \forall\ t, r, e \quad \forall (e,g) \in M_{e2g} \end{aligned} \tag{44}$$

$ProdPe$	production of primary energy	`vm_prodPe`
$FuelEx$	fuel extraction rate of the grade g of an exhaustible resource e	`vm_fuExtr`
$XRes$	energy export	`vm_Xport`
$MRes$	energy import	`vm_Mport`
τ	trade costs (i.e. energy losses)	`p_costsPEtradeMp`
e	energy carrier	`peRicardian(enty)`
$E1$	tradable primary energy carriers	`tradePe`
M_{e2g}	combination of exhaustible primary energy carriers and grade levels	`pe2rlf`

---

[3] τ represents energetic losses here, whereas in case of final good import, τ^G represents monetary costs (see sec. 2.2).

3.7.2 Constraints on energy production from renewable sources

Constraints on secondary energy production from renewable sources (`q_limitProd`)

This equation assigns upper limits π on the *technical potential* of secondary energy production technologies from renewable sources (c).

$$\pi_{r,c,g} \geq \nu_{r,c} \cdot \sigma_{r,c,g} \cdot CapDist_{t,r,c,g} \qquad \forall\ t,r \ \ \forall(c,g) \in M_{c2g} \tag{45}$$

$CapDist$	capacity distribution of renewable technologies c	`v_capDistr`
ν	load factor of technology c	`pm_cf`
σ	scaling of the load factor ν dependent on grade level g	`pm_dataren("nur")`
π	maximal production (according to technology c) of secondary energy from non-exhaustible resource via c, g	`pm_dataren("maxprod")`
c	renewable energy transformation technologies	`teReNoBio`
M_{c2g}	combination of renewable technologies and grade levels	`tese2rlfDistr`

3.8 The Emission Equations

3.8.1 Production and Capture of Emissions (`q_emiTeDetail`)

Emissions of type q result from primary to secondary energy transformation, from secondary to final energy transformation (some air pollutants), or transformations within the chain of CCS steps (Leakage).

The equation describes CO_2 released into the atmosphere and CO_2 captured for storage as two different emission types. In primary to secondary energy transformation processes, both types can be generated.

$$\begin{aligned} Emi_{t,r,c,q} = & \sum_{(p,s,c) \in M_{p2s}} \gamma_{t,r,q,p,s,c} \cdot DemPe_{t,r,p,s,c} \\ & + \sum_{(s,f,c) \in M_{s2f}} \gamma_{t,r,q,s,f,c} \cdot ProdFe_{t,r,s,f,c} \\ & + \sum_{(c,i,i+1,q)} \sum_{(c,g)} \gamma_{t,r,c,q} \cdot CCS_{t,r,i,i+1,c,g} \end{aligned}$$

$$\forall\ t,r \ \ \forall(q,c) \in M_{e2t} \quad (c,i,i+1,q) \in M_{ccs2l} \ \ (c,g) \in M_{c2g} \tag{46}$$

$DemPe$	demand of primary energy	`vm_demPe`
$ProdFe$	production of final energy	`vm_prodFe`
Emi	amount of emissions from type q produced by conversions explained in M_{e2t}	`v_emiTeDetail`
CCS	CO2 emission from transformation in the CCS chain from step i to $i+1$ using technology c with grade level g	`v_co2CCS`
γ	emission of type q per energy flow in the transformation e_{in} into e_{out} using technology te	`pm_emifac`
q	emission type	`emiseng(enty)`
i	step of the CCS process chain	
c	all technologies	`te`
p	all primary energy types	`pety(enty)`
s	all secondary energy types	`sety(enty)`
f	all final energy types	`fety(enty)`
M_{ccs2l}	definition of leakage from CCS transformations	`ccs2Leak`
M_{c2g}	combination of technology and grade levels for CCS	`teccs2rlf`
M_{e2t}	definition of emissions from a transformation	`emi2te`
M_{p2s}	definition of primary to secondary energy transformation	`pe2se`
M_{s2f}	definition of secondary to final energy transformation	`se2fe`

3.8.2 MACs (`q_macBase, q_emiMacSector,q_emiMac`)

Mitigation options that are independent of energy consumption are represented using marginal abatement cost (MAC) curves, which describe the percentage of abated emissions as a function of the costs. Baseline emissions are obtained by three different methods: by source (via emission factors), by econometric estimate, and exogenous. Emissions (Qm_q) are calculated as baseline emissions ($\overline{Q}_q$) times (1 - relative emission reduction (λ_q)). In case of CO_2 from landuse (co2luc), emissions can be negative. To treat these emissions in the same framework, we subtract the minimal emission level ϵ from baseline emissions. This shift factor is then added again when calculating total emissions.

$$
\begin{aligned}
Qm_{t,r,q} = \overline{Q}_{t,r,q} \cdot (1 - s \cdot \lambda_{t,r,q}) & \\
+ \quad \epsilon_{t,r} & \qquad \text{if} \quad q = co2luc \\
& \qquad \forall\ t, r, q
\end{aligned} \tag{47}
$$

$\overline{Q}$	baseline emissions	`vm_macBase`
Qm	emissions	`vm_emiMac`
s	switch to turn MACs on and off	`p_macswitch`
λ	relative emission reduction	`p_macAbatLev`
ϵ	minimal land use change emission level	`f_co2magpietax50`
q	type of emissions	`emiMac`

3.8.3 Total emissions (`q_co2eq`)

Total emissions in CO_2 equivalents are computed based on regional GHG emissions from different sectors j (energy system, non-energy system, exogenous, CDR technologies):

$$Qtot_{t,r} = \sum_j EmiCO2_{t,r,j} + \omega \cdot \sum_j EmiCH4_{t,r,j} + \delta \cdot \sum_j EmiN2O_{t,r,j} \quad \forall\ t, r \qquad (48)$$

$Qtot$	Total regional CO2 equivalent emissions	`vm_co2eq`
$EmiCO2$	CO2 emissions	`vm_emiAll("co2")`
$EmiCH4$	CH4 emissions	`vm_emiAll("ch4")`
$EmiN2O$	N2O emission	`vm_emiAll("n2o")`
ω	conversion factor for 100yr GWP of CH4	`s_tgch4_2_pgc`
δ	conversion factor for 100yr GWP of N2O	`s_tgn_2_pgc`
j	type of emission sector	

3.8.4 The CO_2 emission constraint (`q_emiCap`)

The initial allocation of permits to a region (QP) must cover its emissions Emi plus its permit exports X minus its permit imports M plus its banking B.

$$Qtot_{t,r} + X_{t,r} - M_{t,r} + B_{t,r} \leq QP_{t,r} \quad \forall\ t, r \qquad (49)$$

B	emission permit banking	`vm_banking`
M	emissions permit import	`vm_Mport("perm")`
$EmiTot$	amount of emissions in CO_2eq	`vm_co2eq`
QP	initial permit allocation	`vm_perm`
X	emissions permit export	`vm_Xport("perm")`

3.9 The CCS Equations

CCS Balance (`q_balCCS`)

The right hand side of the equation calculates the total amount of $\mathrm{CO_2}$ captured (Emi) from all relevant processes c that produce captured CO2.[4] This amount enters the CCS process chain (left hand side).

$$\sum_{c \in M_{CCS}} CCS_{t,r,c,i} = \sum_{c \in M_c} Emi_{t,r,c} \quad \forall\, t, r \quad i = 1 \tag{50}$$

Transformation in the CCS chain (`q_transCCS`)

Process steps in the CCS chain are subject to leakage. The amount of captured $\mathrm{CO_2}$ at one step i of the CCS chain is thus the amount of $\mathrm{CO_2}$ in the previous step $i-1$ times 1 minus specific emission coefficient γ that considers the leakage.

$$(1 - \theta_{t,r,c,i}) \cdot CCS_{t,r,c,i} = CCS_{t,r,c,i+1} \qquad \forall\, t, r, i \quad \forall\, c \in M_{CCS} \tag{51}$$

Constraint on CCS injection (`q_limitCCS`)

The storage space for carbon that is injected over time is limited by ψ.

$$\sum_{t} \Delta t \cdot CCS_{t,r,c,i} \leq \psi_r \quad \forall r \quad \forall\, (c,i) \in M_{inj} \tag{52}$$

Emi	amount of captured $\mathrm{CO_2}$ emissions produced by various conversion technologies	`v_emiTeDetail`
CCS	amount of $\mathrm{CO_2}$ in step i of the CCS chain to be transformed in to next one using technology c	`v_co2CCS`
θ	specific $\mathrm{CO_2}$ emissions leakage rate in the CCS chain (default=0)	`pm_emifac`
ψ	maximal cumulative injection for CCS	`p_dataccs("quan")`
c	Technology that produces captured $\mathrm{CO_2}$ in CCS chain at stage i	`te`
i	stage in the CCS chain, i=1,...,4	
M_{CCS}	definition of technologies in CCS chain	`ccs2te`
M_c	definition of technologies that produce captured $\mathrm{CO_2}$	`emi2te`
M_{inj}	definition of CCS injection technologies	`ccs2te2("ico2")`

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[4] Note that "$\mathrm{CO_2}$ captured" is treated as an emission type distinct from $\mathrm{CO_2}$ released into the atmosphere (see sec. 3.8.1 also).

4 Climate Module

4.1 MAGICC (Interface and further reference)

The reduced-form climate model MAGICC calculates the climate system dynamics in response to various emission types given in annual resolution.

For a model description, please refer to Meinshausen, Raper, and Wigley (2011).

MAGICC is coupled to REMIND via emissions of fossil- and other CO_2, CH_4, N_2O, F-Gases, SO_2, BC, OC, NO_x, CO, VOC, and NH_3. The emissions are calculated either endogenously by REMIND, derived from endogenous values during post-processing, carried over from a matching exogenous scenario (e.g. RCP) or a mix of the above.

Emissions are mapped from REMIND regions to the five RCP regions (OECD90, REF, ASIA, MAF, LAM) using the 2005 SO_2 emissions as weighting factors.

Group	Description	Emissions
a	model endogenous emissions	fossil CO_2, other CO_2, CH_4, N_2O
b	emissions partly derived from model endogenous values during post-processing, partly exogenous	SO_2, BC, OC, CO, NO_x, VOC, NH_3
c	exogenous emissions	F-Gases

Emissions of group a and c are simply mapped to RCP regions.

$$EmiMag_{t,r',q} = \sum_r \omega_{r,r'} \; Emi_{t,r,q} \tag{53}$$

Post-processing emissions (group b) are calculated by (and summed over) sectors. Emissions from power production, industry, residential and transport are derived from endogenous activity data. Emissions from fossil fuel extraction, industry processes, solvents, agriculture, agricultural waste burning, forest burning, grassland burning, and waste are carried over from external scenarios.

$$\begin{aligned} &EmiMag_{t,r',q} = \\ \sum_r \omega_{r,r'} \cdot \sum_{s \in S1} \sum_{(s,c) \in M_S} &\left(\left(\sum_{(s,c) \in M_S} EmiP_{t,r,q,s,c} \right) + EmiX_{t,r,q,s} \right) \cdot \sigma_{q,s} \\ &+ \sum_{s \in S2} EmiX_{t,r,q,s} \end{aligned} \tag{54}$$

$EmiMag$	emissions exported to MAGICC	`pm_magicc_emi`
ω	weighting factor mapping Remind to RCP regions	`pm_regi_2_MAGICC_regions`
Emi	endogenous or exogenous emissions	`vm_emiTe` or `vm_emiMac` or `p_emiFgas`
$EmiP$	air pollutant emissions calculated during post-processing	`pm_emi_postrun`
$EmiX$	air pollutant emissions of exogenous sectors	`p11_ef_limits_wp4` or `pm_limits_wp4_rcp`
σ	scaling factor to match 2005 RCP emissions	`pm_scale_rcp`
r'	RCP region	`RCP_regions_world_bunkers`
r	REMIND region	`regi`
q	emission type	`emiRCP`
s	sector	`sector`
c	technology	`te`
S_e	sectors with endogenised emissions	–
S_x	sectors with exogenous emissions	–
M_S	mapping of technologies to sectors	`sectorEndoEmi2te`

Global emissions also include external scenario data for emissions from international aviation and shipping.

Emissions are exported to MAGICC in native REMIND time steps, except for a spin-up interval from 2000 to 2005, where emissions are interpolated linearly to ensure a smooth transition between historical and model data.

In policy experiments, MAGICC can be used to iteratively adapt emission budgets or price levels to meet forcing or temperature targets. To that end, the total anthropogenic radiative forcing (file `DAT_TOTAL_ANTHRO_RF.OUT`) is read from MAGICC into REMIND.

5 Optimization

Two solution concepts calculating the Pareto-optimal solution are implemented: Negishi, and Nash procedures, as described in detail in this section[5].

The Negishi procedure uses a joint-optimization method to find the cooperative solution between regions. The Nash procedure uses a Walrasian-auctioneer mechanism to coordinate regional trade, and arrives at the non-cooperative solution.

Non-internalized inter-regional externalities drive a wedge between the cooperative and the non-cooperative solution – global technological learning-by-doing is the only externality in the default model version.

Major element of the solution algorithms are prices p_j. They always represent net present value prices. Variables, parameters, sets and set elements used in both of the two approaches are:

M_j^i	imports of commodity j in iteration i	`vm_Mport`
X_j^i	exports of commodity j in iteration i	`vm_Xport`
p_j^i	Prices on commodity markets j in iteration i	`pm_pvp`
g	composite good	`"good"`
q	emission permits	`"perm"`
i	iteration	`iteration`
j	traded commodities	`trade`
C1 = {C2,g,q}	traded commodities	
C2	traded energy types	

5.1 Negishi procedure

Within this solution approach, the objective functions of the individual regions are merged to a global objective function (see Eq. 1) by means of welfare weights W. The Negishi procedure adjusts the welfare weights in an iterative process around the model optimization.

- A distinguished Pareto-optimal solution, which without non-internalized externalities also corresponds to a market solution, is obtained by adjusting the welfare weights iteratively according to the intertemporal trade balances B^i:

$$\overline{B}_r^i = \sum_t \sum_{j \in \mathrm{C1}} \Delta t \cdot p_{t,j}^i \cdot (X_{t,r,j}^i - M_{t,r,j}^i) \ \forall\ r, i \tag{55}$$

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[5] Both methods are described in full detail in Leimbach et al. (2015).

The intertemporal trade balance B of each region is the net present value of all trade flows to and from this region, and is computed as the sum of net trade volumes of all tradable entities evaluated by associated shadow prices. i is the iteration step.

- Shadow prices p of each tradable entity are determined from the marginal values of the associated trade balance (see sec. 2.7).

$$p_{t,j} = \left| \frac{\partial U}{\partial (\sum_r (X_{t,r,j} - M_{t,r,j}))} \right| \quad \forall\ t, j \neq q \tag{56}$$

In case of permits, the maximum marginal of the permit trade balance (see sec. 3.8.4) and the emission summation is considered:

$$p_{t,j} = \max \left(\left| \frac{\partial U}{\partial (\sum_r (X_{t,r,j} - M_{t,r,j}))} \right|, \left| \frac{\partial U}{\partial (\sum_r (Q_{t,r} + Qneg_{t,r} - Qtot_{t,r}))} \right| \right) \quad \forall\, t, j = q$$

- A new set of welfare weights is derived iteratively:

$$\overline{W}_r^{i+1} = \overline{W}_r^{i} + \frac{\overline{B}_r^{i}}{\sum_t ((1+\rho_r)^{-t}\, \overline{L}_{t,r})} \quad \forall\ r, i \tag{57}$$

- The global sum of all regional intertemporal imbalances ω is an indicator for the convergence of the solution (is close to zero in the final solution):

$$\omega^i = \sum_r \left| \overline{B}_r^{i} \right| \qquad \forall\ i \tag{58}$$

$Qtot$	total GHG emissions	`v_co2eq`
$Qneg$	non-energy related GHG emissions	`vm_emiMac`
Q	energy related GHG emissions	`vm_emiTe`
$\overline{W}$	Negishi weights	`p80_nw`
$\overline{B}$	intertemporal trade balance deficit	`p80_defic`
$\overline{L}$	population	`pm_pop`
ρ	pure rate of time preference	`pm_prtp`
ω	global sum of intertemporal trade deficits and surpluses	`p80_defic_sum`

5.2 Nash procedure

The Nash procedure computes an inter-temporal and inter-regional equilibrium among independent regions - which coincides with the Negishi equilibrium in the absence of inter-regional externalities. Currently, the only inter-regional externality in the model is learning-by-doing in the energy sector (see Sec. 3.6).

Regional social planner models, each including the inter-temporal trade balance, are solved in parallel, and choose their trade patterns for given prices. The Nash algorithm then computes surpluses on all markets, and adjusts prices in order to reduce market surpluses in the next iteration. This procedure is then iterated until residual market clearances are reasonably small.

5.2.1 Inter-temporal budget equation (`q80_budg_intertemp`)

The inter-temporal trade balance is fulfilled by construction in each region:

$$0 = \sum_t \Delta t \sum_{j \in \mathrm{C1}} p^i_{t,j} \left(1 + \mathrm{An}^i_{t,r,j}\right) \left(X^i_{t,r,j} - M^i_{t,r,j}\right) \quad \forall\ r, i \tag{59}$$

The anticipation term $\mathrm{An}_(r, t, j)$ is a helper construct, which does not influence the solution point. It enables regions to anticipate price changes on the market in response to their trade decisions *within* the optimization, helping the solution to converge. During the iteration, as soon as trade deficits are reasonably small, this term is faded out and thus does not influence the solution.

5.2.2 Regularization (adjustment costs) (`q80_costAdjNash`)

This equation is a helper construct to aid the convergence process. Deviation from the trade pattern of the previous iteration are penalized with quadratic adjustment costs, which are accounted for in the regional budget equation. This regularization helps the convergence process as it prevents quickly diverging markets, but does not influence the solution point. Adjustment costs are priced into the budget equation, and calculated using a weighting parameter ν_j according to:

$$\mathrm{AdjNa}^i_{t,r} = \sum_{j \in \mathrm{C1}} \frac{p^i_{t,j}\, \nu^i_j}{\overline{No}^i_j} \left(X^i_{t,r,j} - M^i_{t,r,j} - \left(X^{i-1}_{t,r,j} - M^{i-1}_{t,r,j}\right)\right)^2 \quad \forall\ t, r \tag{60}$$

5.2.3 Iterative price adjustment algorithm

After each successful round of regional optimizations, this algorithm calculates prices for the next iteration $i + 1$ from the surplus on all markets:

- Trade deficits $\overline{S}$ of each tradable commodity j are calculated from imports M and exports X:

$$\overline{S}^i_{t,j} = \sum_r (X^i_{t,r,j} - M^i_{t,r,j}) \quad \forall\ t, j \in \mathrm{C1} \tag{61}$$

- From these surpluses, the price for the next iteration is calculated:

$$p^{i+1}_{t,j} = p^i_{t,j} \left(1 - \eta^i_j \frac{\overline{S}^i_{t,j}}{\overline{No}^i_j}\right) \quad \forall\ t, j \in \mathrm{C1} \tag{62}$$

The parameter $\overline{No}_j^i$ normalizes to a proxy for the potential volume of the corresponding market j [6].

- This procedure is iterated until market surpluses are reasonably small. At that point though, the price anticipation terms $\mathrm{An}_{t,r,j}$ are still non-zero, which influences the solution point, as it gives regions market power. Thus, the anticipation terms are now faded out, while iterating further.

5.2.4 Convergence indicators

Iterations stop once residual deviations from market clearances fall below the threshold ϵ:

$$\left|\overline{S}_{t,j}^i\right| < \epsilon \qquad \forall\, t, j \tag{63}$$

The net present value of these residual deviations clearances volumes `p80_defic_sum` is a useful single-number indicator for the convergence of the solution algorithm.

$$\overline{D}^i = \sum_t \sum_j \Delta t \cdot p_{t,j}^i \cdot \overline{S}_{t,r,j}^i \quad \forall i \tag{64}$$

$AdjNa$	Adjustment costs	`vm_costAdjNash`
η	parameter of price adjustment	`p80_etaXp`, `p80_etaLT`, `p80_etaST`
ν	weighting facor	`p80_etaAdj`
ϵ	tolerance level	`p80_surplusMaxTolerance`
$\overline{S}$	trade deficit	`p80_surplus`
$\overline{No}$	normalization parameter	`p80_normalize0`
$\overline{D}$	deviation from market clearance	`p80_defic_sum`

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[6] The parameter η_j includes a time-dependent price correction. This – heuristic – correction helps energy markets converge faster.

6 Literature

Leimbach et al.(2015): Leimbach, M., Schultes, A., Baumstark, L., Giannousakis, A., Luderer, G. (2017): Solution algorithms of large-scale Integrated Assessment models on climate change. Annals of Operations Research 255, 29-45. doi:10.1007 s10479-016-2340-z.

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