A Very Brief and Shallow Introduction to: Complexity, Chaos, and Fractals

J. Kropp

Other Possible Titles:

Chaos for Dummies

Learn Chaos in 1 hour

All you need to know about Chaos

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Topics

- Definition of Complexity
- Chaos Theory – What is it really?
  Background, Little Theory and Examples
- Fractals – What is it really?
  Background and History
- Relevance of Complexity, Chaos, and Fractals to applications

There is no common definition for complexity….

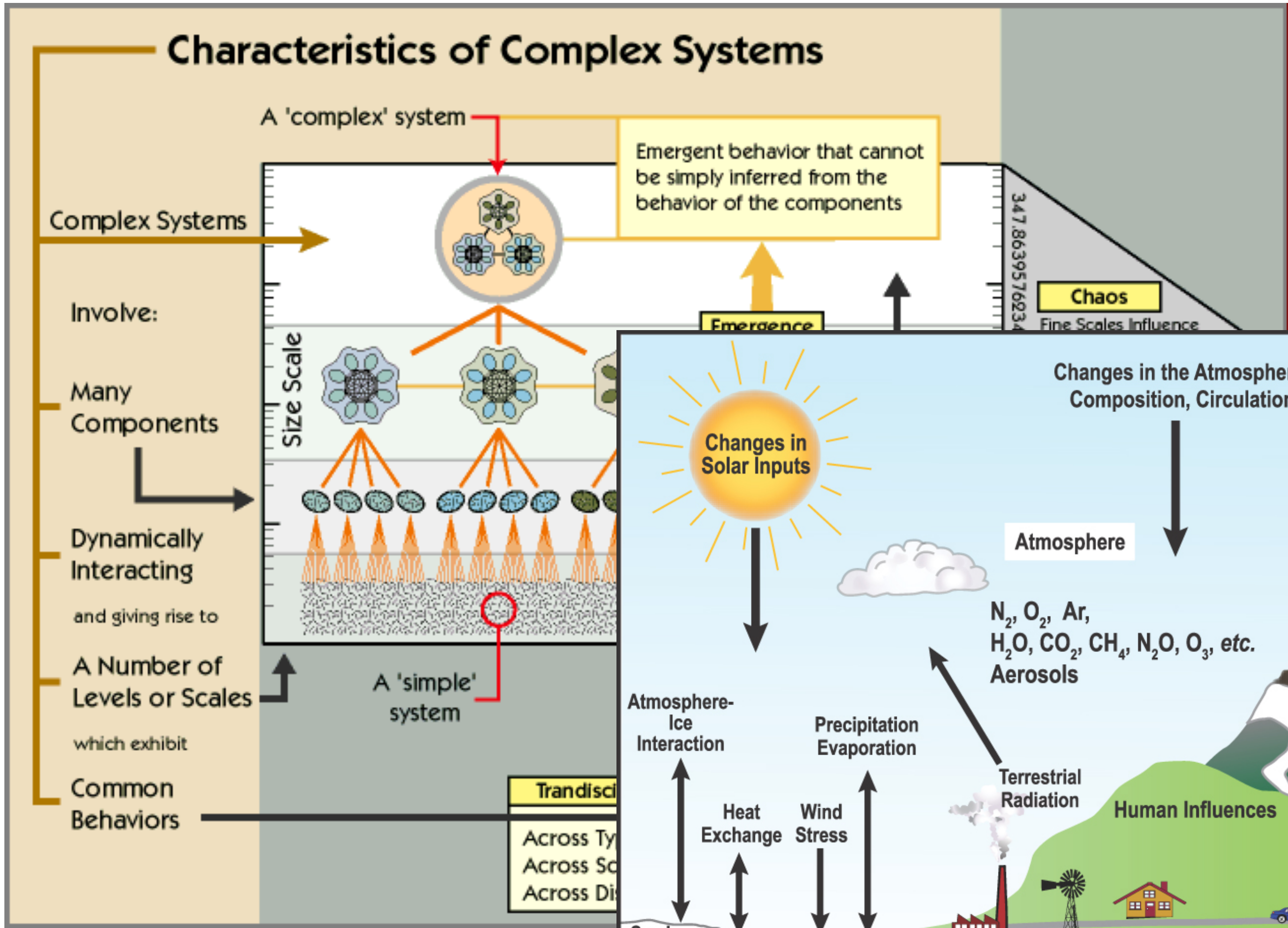
- Genetically derived from studies:
  - A system is complex when it is composed of **many parts** that interconnect in **intricate ways**. (Joel Moses, "Complexity and Flexibility"). This definition has to do with the number and nature of the interconnections. Metric for intricateness is **amount of information** contained in the system
  - A system presents **dynamic complexity** when **cause and effect are subtle**, over time. (Peter Senge, "The Fifth Discipline"). E.g.: dramatically different effects in, the short-run and the long-run; dramatically different effects locally and in other parts of the system; obvious interventions produce non-obvious consequences
  - A system is complex when it is composed of a group of related units (subsystems), for which the **degree and nature of the relationships is imperfectly known**. (Joseph Sussman, "The New Transportation Faculty"). The overall **emergent behavior** is difficult to predict, even when subsystem behavior is readily predictable. Small changes in inputs or parameters may produce large changes in behavior
  - **Complexity theory** and **chaos theory** both attempt to reconcile the unpredictability of non-linear dynamic systems with a sense of underlying **order and structure**. (David Levy, "Applications and Limitations of Complexity Theory in Organizational Theory and Strategy"). Implications: pattern of short-term predictability but long-term planning impossible, dramatic change unexpectedly, organizations can be tuned to be more **innovative and adaptive**

Chaos Theory

- Dictionary Meaning of *Chaos* – "*a state of things in which chance is supreme; especially **:** the confused unorganized state of primordial matter before the creation of distinct forms*" (Webster).
- Chaos Theory represents a big jump from the way we have thought in the past – a paradigm shift.
- Traditional notion of chaos – *unorganized, disorderly, random* etc.

- *NB! Chaos Theory has nothing do with this traditional notion per se!*

Views of Complex Systems



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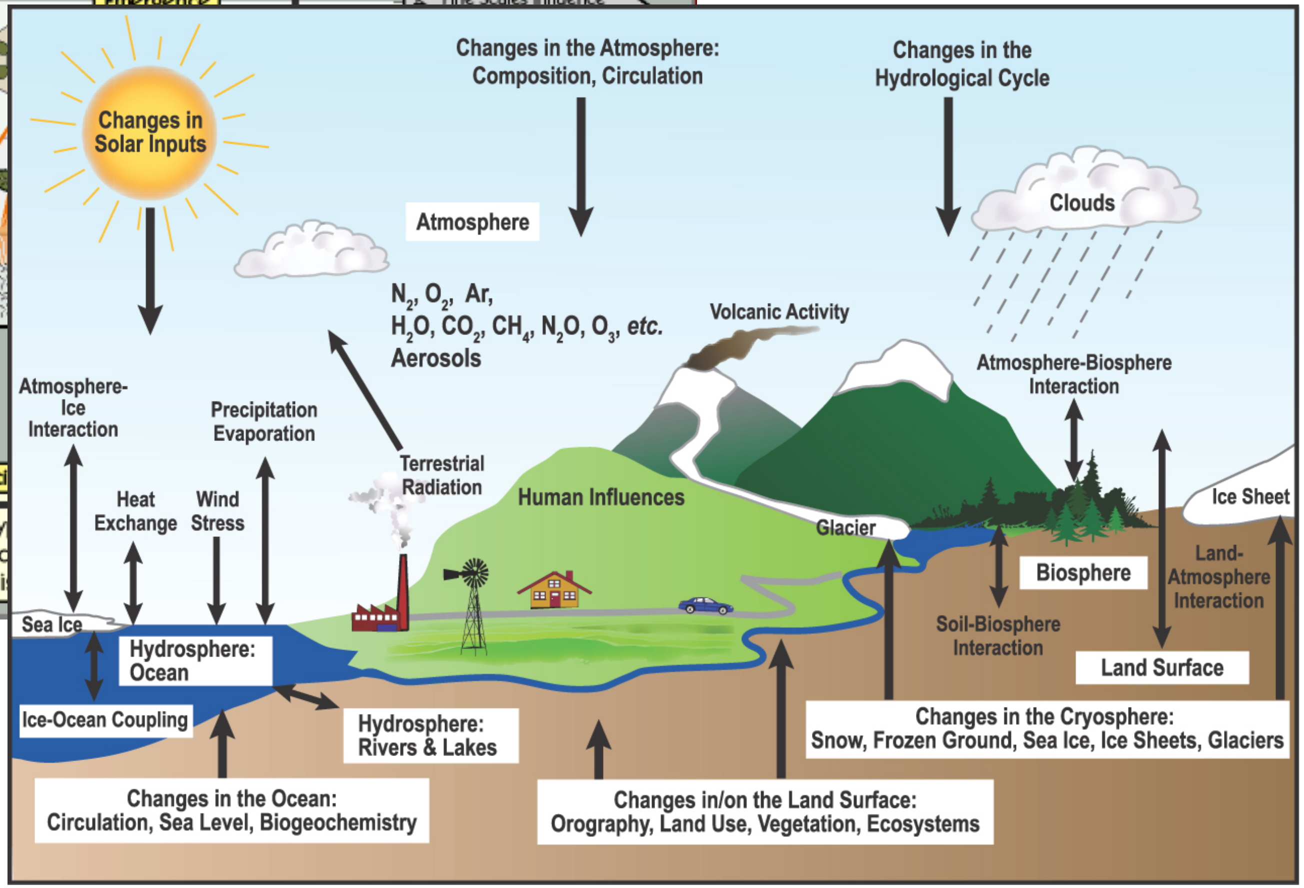


The Debate…

- Classical and Modern notions of Determinism – Classical Mechanics Vs. Quantum Mechanics. Sir Isaac Newton initiated Classical Mechanics that lasted (still is) 300 years.
- Not everything can be observed or predicted perfectly or in a deterministic (very accurate) fashion.
- Examples: At atomic scale, everything gets probabilistic (Throwing a ball and an atom – motion is wave-like De-Broglie's motion)
- Uncertainties in various sources causes our observations and predictions to behave randomly.
- But then Chaos Theory comes in and bridges the gap

Not all the randomness we see is really due to chance, it could well be due to 'deterministic' factors

Definition of ‘chaos’

- Chaos: Mathematically, this term is used to describe dynamical systems in which small changes in initial conditions lead to large changes in the solution after some period of time

Examples of chaotic systems:

- – The weather (the ‘Butterfly effect’)
- – Double pendulum
- – The geo-dynamo (or solar dynamo)
- – The inner solar system
- – The climate system

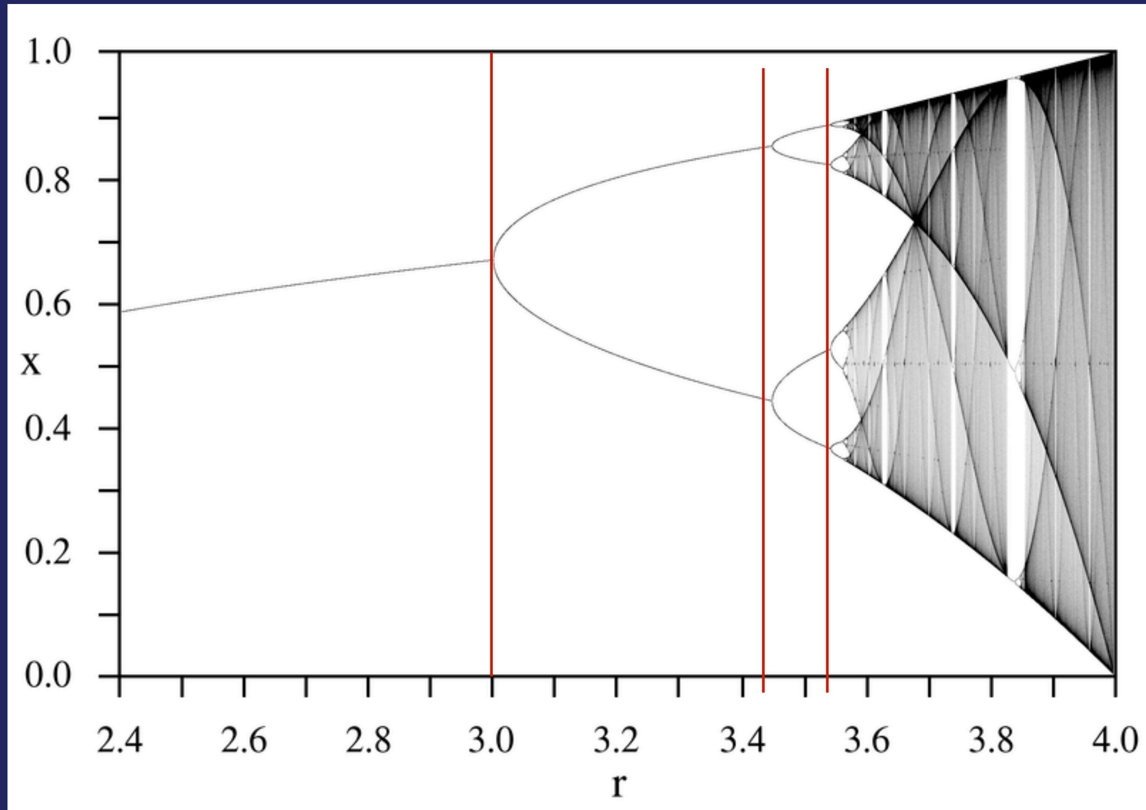
Approach

- **M. Feigenbaum** studies of population growth models (Logistic Equation)

$$\text{Population}_t = \text{GrowthRate}*\text{Population}_{t-1}(1-\text{Population}_{t-1})$$

Feigenbaum Constant:
4.6692016…

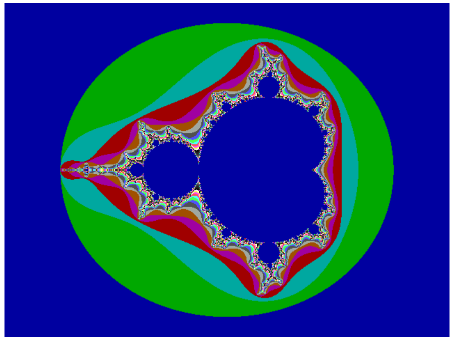
Relation between bifurcation intervals



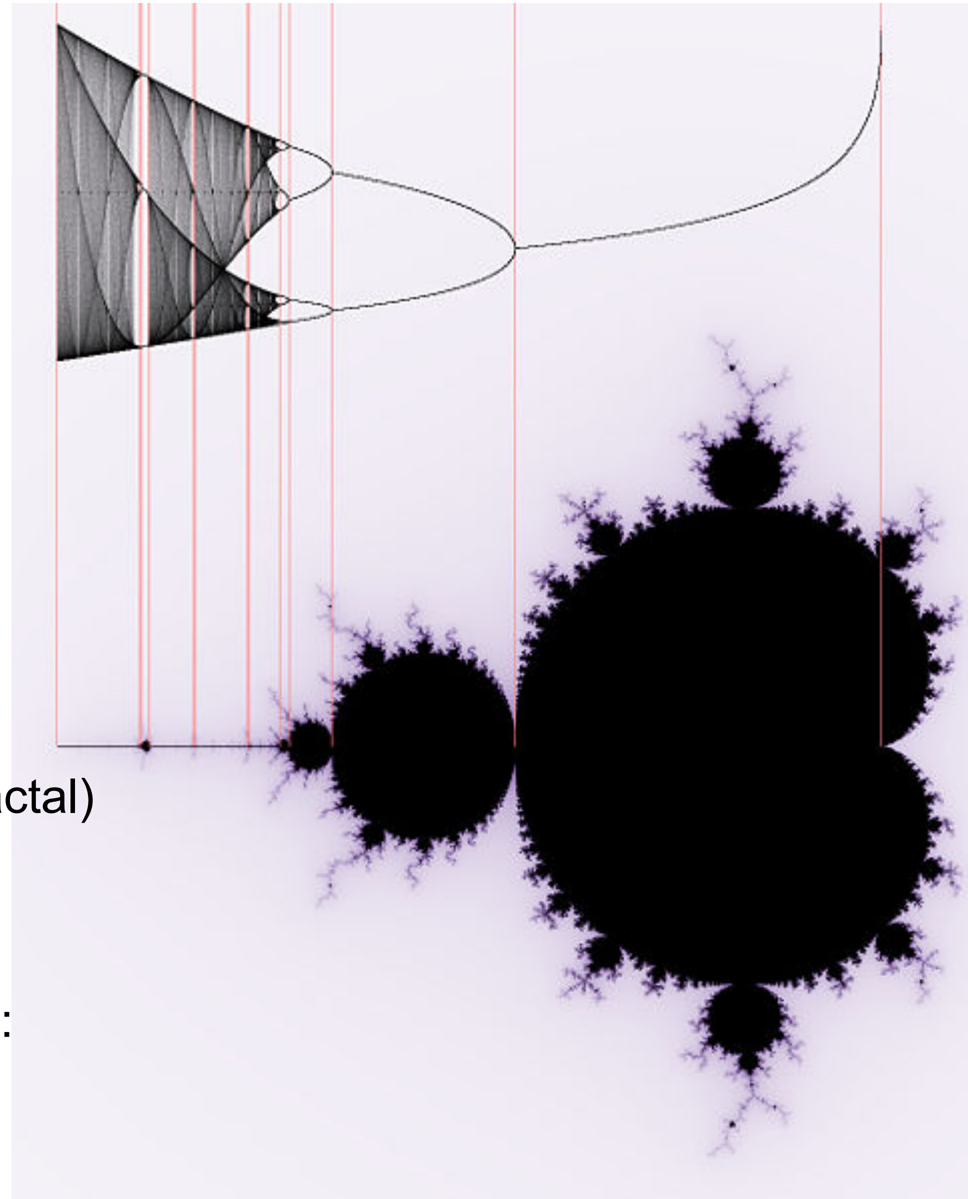
Growth Rate

Relation Chaos & Fractals

- The Feigenbaum constant appears in many other contexts
- the **Mandelbrot Set (Extension to complex numbers)**
  - Equation: $Z(n+1)=Z(n)^2+C$, C and Z imaginary numbers
  - Mapping: represents the number of
  - iterations need for $|Z(n)|>2$



Apfelmännchen (Fractal)



The importance of the Feigenbaum constant:
…..it is an **invariant**

Summing Up (for your rebound….)

- Chaos – is about the deterministic factors (non-linear relationships) that cause things to look random
- A unique property that define a 'Chaotic System'
- Sensitivity to initial conditions – causing large divergence in the prediction. But this divergence is not infinite, it oscillates within bounds.
- Discovered by Ed Lorenz in Weather Modeling

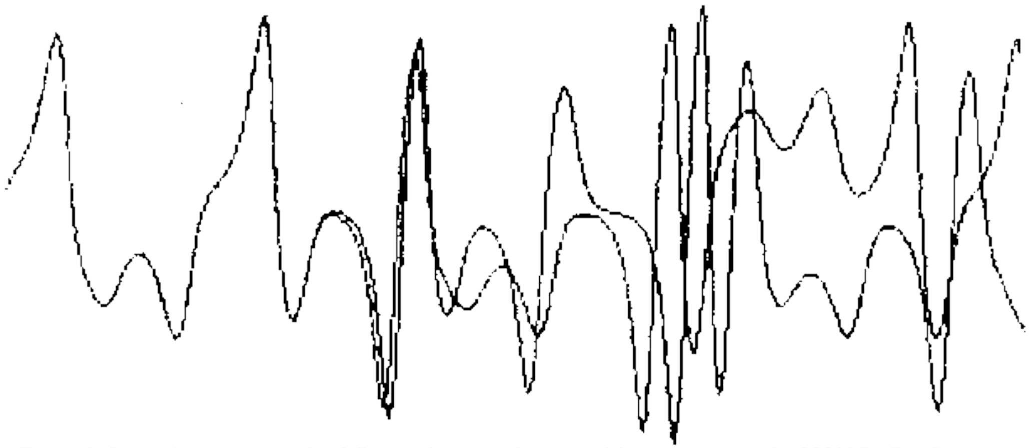
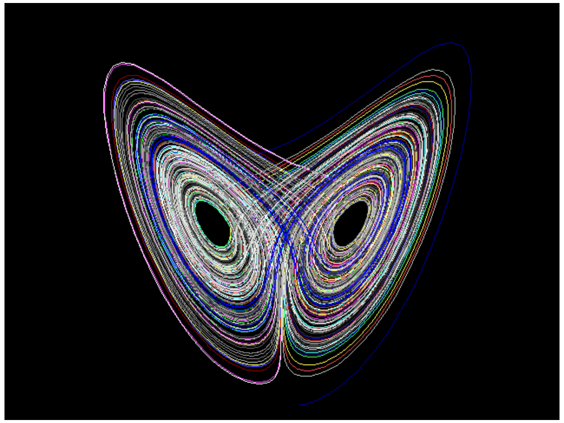


Figure 1: Lorenz's experiment: the difference between the start of these curves is only .000127. (Ian Stewart, Does God Play Dice? The Mathematics of Chaos, pg. 141)



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- Other Analogies – Think about the 100m sprint at the Olympics. Sprinters all start the same (supposedly the same initial conditions and they are all the best). Yet, one tiny change (like a draught of air, failing to hear/respond the whistle on time can cost them a medal.

- Or life itself – more chaotic. One tiny decision you take today (apparently tiny), you have no idea where it might take you in the long after an accumulation of the triggering effects.

- Butterfly Effect: The flapping of a single butterfly's wing today produces a tiny change in the state of the atmosphere. Over a period of time, what the atmosphere actually does diverges from what it would have done. So, in a month's time, a tornado that would have devastated the Indonesian coast doesn't happen. Or maybe one that wasn't going to happen, does. (Ian Stewart, *Does God Play Dice? The Mathematics of Chaos*, pg. 141)

Fractals – they are about *Scale*

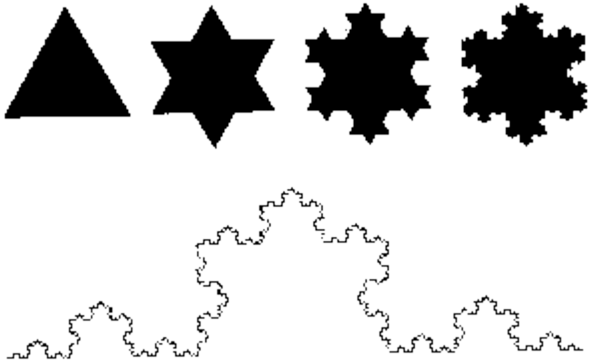
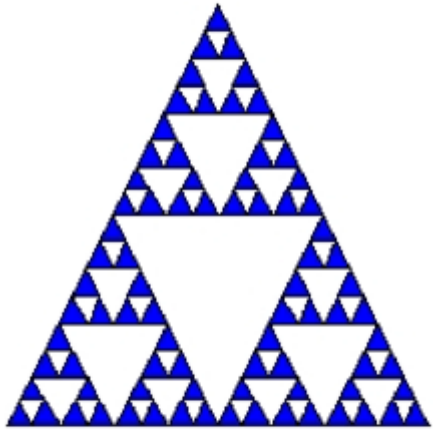
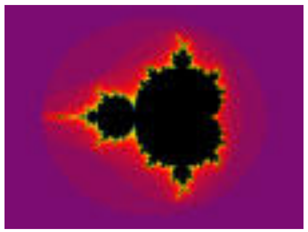


Figure 4: the Koch curve (James Gleick, Chaos - Making a New Science, pg. 99)





Discovered by B. Mandelbrot (Mandelbrot Set)

It's all about scales and its invariance (not just space though – can also time

And self-organized similarity (scale invariance) a rather new term coined these days

- Another manifestation of Chaos theory in the form of scales

More Examples of Fractals

- Look at Clouds and Mountains
- River Networks
- Coastlines

- Fractal geometry Vs Euclidean Geometry – the debate. Fractals allow us to be more realistic

Coast of England Problem

- How small is small to define ‘precision’ to measure an entity (say length, area, volume)?
- If precision increases, and the measurements are done again, then will new measurements go on increasing indefinitely ? Say 50 years ago we could measure length upto 1 micron. Now we have a device to measure up 1 nanometer. So, if we measured the coast of England by the ever increasingly precise measuring instrument will the coast length increase indefinitely?
- Very similar to the Zeno’s paradox we discussed in first class. The rabbit and the hare starting off a race.

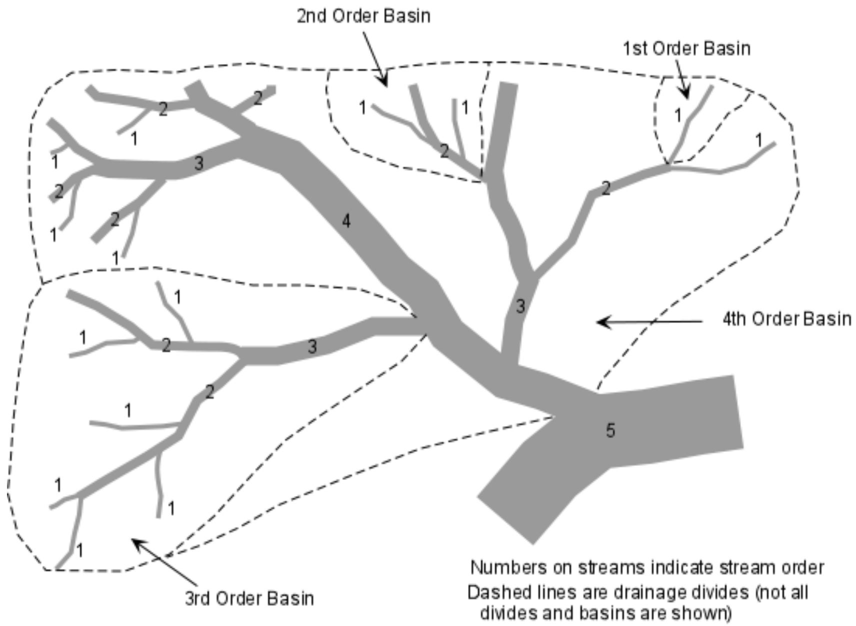
Mandelbrot asked the same question: What is the coast of England? For a specific reason.

Real-life examples of Fractals (in space)

Are there any straight lines, boxes, rectangles based on Euclidean geometry?

Yet, these shapes seem to be repeating – Nature is more fractal and we humans have traditionally tried to model it in our paradigms of 'straight lines' and adding too much order

River Drainage Network – Very Fractal



Okay, that's all about Complexity, Chaos and Fractals, but so what for our sciences?

- Let's start a discussion – How can we use chaos theory and fractals to better understand sciences on natural systems, do better prediction, estimation forecasting ?
- Opportunities may be endless but yet to be fulfilled.
- Chaos Theory can help 'tame' complexity, uncertainty (that we think as random) and make them more 'modelable' (less uncertain)
- Spatial patterns and Temporal patterns can be better predicted by looking at the fractal nature – rainfall is fractal is space (if clouds are), so its spatial distribution can be modeled 'better'.
- E.g. watershed's response to rainfall – initial conditions and chaos.
- Remember – Chaos Theory is 'physical' based on logic (deterministic equations) unlike stochastic theory.

Have nice holidays!