Model Equations MICA
Model of International Climate Agreements
Kai Lessmann∗† Ulrike Kornek†‡
March 31, 2016

Model Equations

In the following listing, \( t \) refers to time, \( i \) refers to regions.

Preferences

Social welfare of region \( i \)

\[
W_i = \int_0^\infty n_i t U(c_{it}/n_{it}) e^{-\rho t} dt \quad (0.1)
\]

Instantaneous utility

\[
U(c_{it}/n_{it}) = \begin{cases} 
\frac{(c_{it}/n_{it})^{1-\eta}}{1-\eta} & \text{if } \eta \neq 1 \\
\log(c_{it}/n_{it}) & \text{if } \eta = 1.
\end{cases} \quad (0.2)
\]

Technology

Economic output net of abatement costs and climate change damages

\[
y_{it} = (1 - \Lambda_{it} - \Omega_{it}) F(l_{it}, k_{it}) \quad (0.3)
\]

Production technology

\[
F(l_{it}, k_{it}) = \alpha_0 y_{i0} \left[ (1 - \gamma) \left( \frac{\lambda_{it} l_{it}}{\lambda_{i0} l_{i0}} \right)^{\rho_F} + \gamma \left( \frac{k_{it}}{k_{i0}} \right)^{\rho_F (1/\rho_F)} \right] \quad (0.4)
\]

∗Potsdam-Institute for Climate Impact Research (PIK), PO Box 60 12 03, D-14412 Potsdam, Germany
†Mercator Research Institute on Global Commons and Climate Change (MCC), Torgauer Str. 12-15, 10829 Berlin, Germany
Accumulation of capital, initially $k_{i0}$

$$\frac{d}{dt} k_{it} = i_{it} - \delta_{i} k_{it}$$  \hspace{1cm} (0.5)

**Emissions and Emission Allowances**

Emissions as a byproduct of production, reduced by emission intensity and abatement effort

$$e_{it} = y_{it} \sigma_{it} (1 - a_{it})$$  \hspace{1cm} (0.6)

Abatement costs

$$\Lambda_{it} = b_{1}^{1} \cdot (a_{it})^{b_{2}}$$  \hspace{1cm} (0.7)

All emissions are covered by allowances net of allowance exports.

$$e_{it} \leq q_{it} - z_{it}$$  \hspace{1cm} (0.8)

Trade in allowances is balanced in every time period.

$$\sum_{j} z_{jt} = 0, \hspace{1cm} \forall t$$  \hspace{1cm} (0.9)

**Climate Dynamics**

CO2 concentration changes with total allowances (same as total emissions), initially $C_{0}$.

$$\frac{d}{dt} C_{t} = \zeta Q_{t} - \kappa (C_{t} - C_{0}) + \psi E_{t}$$  \hspace{1cm} (0.10)

Definition of global total of emission allowances

$$Q_{t} = \sum_{i} q_{it}$$  \hspace{1cm} (0.11)

Global emissions stock, initially $E_{0}$, rises with per period total allowances.

$$\frac{d}{dt} E_{t} = Q_{t}$$  \hspace{1cm} (0.12)

Temperature change, initially $T_{0}$, is determined by CO2 concentration.

$$\frac{d}{dt} T_{t} = \mu \log(C_{t}/C_{0}) - \phi (T_{t} - T_{0})$$  \hspace{1cm} (0.13)

Climate change damages

$$\Omega_{it} = \theta_{2i}(T_{t})^{2}$$  \hspace{1cm} (0.14)
Budget constraints

Budget constraint of the Ramsey household

\[ y_{it} + m_{it} = c_{it} + i_{it} + x_{it} \]  \hspace{1cm} (0.15)

Intertemporal budget constraint for trade in goods and allowances

\[ \int_0^\infty p_t m_{it} \, dt = \int_0^\infty p_t x_{it} + p_t^z z_{it} \, dt \]  \hspace{1cm} (0.16)

Parameters and Variables

Parameters

- \( \alpha_{it} \) = Total factor productivity
- \( \gamma \) = Share parameter
- \( \delta_i \) = Rate of depreciation
- \( \zeta \) = Emission to concentration conversion factor
- \( \eta \) = Elasticity of marginal utility
- \( \theta_{i1,i2} \) = Damage function exponent
- \( \kappa \) = Rate of ocean CO2 uptake
- \( \lambda_{it} \) = Labor efficiency
- \( \mu \) = Radiative temperature driving factor
- \( \nu_{i1,i2} \) = Exogenous decarbonization parameters
- \( \rho \) = Pure rate of time preference
- \( \sigma_{it} \) = Exogenous emission intensity improvement
- \( \rho^F \) = Elasticity parameter of production function
- \( \phi \) = Temperature damping factor
- \( \psi \) = Atmospheric retention factor
- \( b_{it}^1 \) = Abatement cost coefficient
- \( b_{it}^2 \) = Abatement cost exponent
- \( C_0 \) = Initial concentration
- \( E_0 \) = Initial cumulative emissions
- \( k_{i0} \) = Initial capital stock
- \( l_{it} \) = Exogenous labor supply

Variables

- \( n_{it} \) = Population number
- \( l_{i0} \) = Initial labor supply
- \( T_0 \) = Initial temperature change
- \( q_{i0} \) = Initial allowances
- \( y_{i0} \) = Initial economic output
- \( a_{it} \) = Abatement
- \( c_{it} \) = Consumption
- \( C_t \) = Carbon concentration in the atmosphere
- \( e_{it} \) = CO2 emissions
- \( E_t \) = Global total emissions
- \( i_{it} \) = Investment
- \( k_{it} \) = Capital
- \( m_{it} \) = Imports
- \( p_t \) = Price of goods
- \( p_t^i \) = Price of allowances
- \( q_{it} \) = Emission allowances
- \( Q_t \) = Cumulative total emission allowances
- \( T_t \) = Global mean atmospheric temperature increase
- \( W_t \) = Intertemporal welfare
- \( x_{it} \) = Exports of region \( i \)
- \( y_{it} \) = Economic output
- \( z_{it} \) = Export of emission allowances
- \( \Lambda_{it} \) = Abatement costs
- \( \Omega_{it} \) = Climate change damages

3