Model Equations MICA

Model of International Climate Agreements

Kai Lessmann[*†] Ulrike Kornek[†*]

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Model Equations

In the following listing, t refers to time, i refers to regions.

Preferences

Social welfare of region i

$$W_i = \int_0^\infty n_{it}\, U(c_{it}/n_{it})\, e^{-\rho t}\, \mathrm{dt} \tag{0.1}$$

Instantaneous utility

$$U(c_{it}/n_{it}) = \begin{cases} \dfrac{(c_{it}/n_{it})^{1-\eta}}{1-\eta} & \text{if } \eta \neq 1 \\ \log(c_{it}/n_{it}) & \text{if } \eta = 1. \end{cases} \tag{0.2}$$

Technology

Economic output net of abatement costs and climate change damages

$$y_{it} = (1 - \Lambda_{it} - \Omega_{it})\, F(l_{it}, k_{it}) \tag{0.3}$$

Production technology

$$F(l_{it}, k_{it}) = \alpha_{it} y_{i0} \left[(1-\gamma) \left(\frac{\lambda_{it} l_{it}}{\lambda_{i0} l_{i0}} \right)^{\rho_F} + \gamma \left(\frac{k_{it}}{k_{i0}} \right)^{\rho_F} \right]^{(1/\rho_F)} \tag{0.4}$$

[*]Potsdam-Institute for Climate Impact Research (PIK), PO Box 60 12 03, D-14412 Potsdam, Germany
[†]Mercator Research Institute on Global Commons and Climate Change (MCC), Torgauer Str. 12-15, 10829 Berlin, Germany

Accumulation of capital, initially k_{i0}

$$\frac{d}{dt}k_{it} = i_{it} - \delta_i k_{it} \tag{0.5}$$

Emissions and Emission Allowances

Emissions as a byproduct of production, reduced by emission intensity and abatement effort

$$e_{it} = y_{it}\,\sigma_{it}\,(1 - a_{it}) \tag{0.6}$$

Abatement costs

$$\Lambda_{it} = b_{it}^{1} \cdot (a_{it})^{b_i^2} \tag{0.7}$$

All emissions are covered by allowances net of allowance exports.

$$e_{it} \leq q_{it} - z_{it} \tag{0.8}$$

Trade in allowances is balanced in every time period.

$$\sum_j z_{jt} = 0, \qquad \forall t \tag{0.9}$$

Climate Dynamics

CO2 concentration changes with total allowances (same as total emissions), initially C_0.

$$\frac{d}{dt}C_t = \zeta Q_t - \kappa(C_t - C_0) + \psi E_t \tag{0.10}$$

Definition of global total of emission allowances

$$Q_t = \sum_i q_{it} \tag{0.11}$$

Global emissions stock, initially E_0, rises with per period total allowances.

$$\frac{d}{dt}E_t = Q_t \tag{0.12}$$

Temperature change, initially T_0, is determined by CO2 concentration.

$$\frac{d}{dt}T_t = \mu \log(C_t/C_0) - \phi(T_t - T_0) \tag{0.13}$$

Climate change damages

$$\Omega_{it} = \theta_{2i}(T_t)^2 \tag{0.14}$$

Budget constraints

Budget constraint of the Ramsey household

$$y_{it} + m_{it} = c_{it} + i_{it} + x_{it} \tag{0.15}$$

Intertemporal budget constraint for trade in goods and allowances

$$\int_0^\infty p_t m_{it} \, \mathrm{dt} = \int_0^\infty p_t x_{it} + p_t^z z_{it} \mathrm{dt} \tag{0.16}$$

Paramters and Variables

Parameters

α_{it} = Total factor productivity
γ = Share parameter
δ_i = Rate of depreciation
ζ = Emission to concentration conversion factor
η = Elasticity of marginal utility
θ_{i2} = Damage function exponent
κ = Rate of ocean CO2 uptake
λ_0 = Intial labor efficiency
λ_{it} = Labor efficiency
μ = Radiative temperature driving factor
$\nu_{i1,i2}$ = Exogenous decarbonization parameters
ρ = Pure rate of time preference
σ_{it} = Exogenous emission intensity improvement
ρ^F = Elasticity parameter of production function
ϕ = Temperature damping factor
ψ = Atmospheric retention factor
b_{it}^1 = Abatement cost coefficient
b_{it}^2 = Abatement cost exponent
C_0 = Initial concentration
E_0 = Initial cumulative emissions
k_{i0} = Initial capital stock
l_{it} = Exogenous labor supply
$n_i t$ = Population number
l_{i0} = Initial labor supply
T_0 = Initial temperature change
$q_i 0$ = Initial allowances
$y_i 0$ = Initial economic output

Variables

a_{it} = Abatement
c_{it} = Consumption
C_t = Carbon concentration in the atmosphere
e_{it} = CO2 emissions
E_t = Global total emissions
i_{it} = Investment
k_{it} = Capital
m_{it} = Imports
p_t = Price of goods
p_t^z = Price of allowances
q_{it} = Emission allowances
Q_t = Cumulative total emission allowances
T_t = Global mean atmospheric temperature increase
W_i = Intertemporal welfare
x_{it} = Exports of region i
y_{it} = Economic output
z_{it} = Export of emission allowances
Λ_{it} = Abatement costs
Ω_{it} = Climate change damages